GATE CLOUD

SIGNALS & SYSTEMS

Sample Chapter
GATE CLOUD Signals & System by Kanodia

GATE CLOUD

SIGNALS & SYSTEMS

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Jaipur

GATE CLOUD Signals & Systems, 1e
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Preface to First Edition

GATE Question Cloud caters a versatile collection of Multiple Choice Questions to the students who are preparing for GATE(Gratitude Aptitude Test in Engineering) examination. This book contains over 1500 multiple choice solved problems for the subject of Signals & Systems, which has a significant weightage in the GATE examinations of EC, EE & IN branches. The GATE examination is based on multiple choice problems which are tricky, conceptual and tests the basic understanding of the subject. So, the problems included in the book are designed to be as exam-like as possible. The solutions are presented using step by step methodology which enhance your problem solving skills.

The book is categorized into eleven chapters covering all the topics of syllabus of the examination. Each chapter contains :

- Exercise 1 : **Theoretical & One line Questions**
- Exercise 2 : **Level 1**
- Exercise 3 : **Level 2**
- Exercise 4 : **Mixed Questions taken form previous examinations of GATE & IES.**
- Detailed Solutions to Exercise 2, 3 & 4
- Summary of useful theorems

Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge the comments, criticism and suggestion from the users of this book which leads to some improvement.

Wish you all the success in conquering GATE.

Authors

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CHAPTER 1

CONTINUOUS TIME SIGNALS

EXCERCISE 1.1

MCQ 1.1.1 The graphical representation of a signal in the time domain is known as

(A) frequency (B) waveform

(C) frequency spectrum (D) none of the above

MCQ 1.1.2 A continuous-time signal is a signal in which the independent variable is

(A) discrete (B) continuous

(C) (A) or (B) (D) none of the above

MCQ 1.1.3 Digital signals are those signal which

(A) do not have a continuous set of values

(B) have values at discrete instants

(C) can utilize decimal or binary system

(D) are all of the above

MCQ 1.1.4 A deterministic signal is the signal which

(A) can not be represented by a mathematical expression

(B) has no uncertainty

(C) has uncertainty

(D) none of the above

MCQ 1.1.5 A random signal is the signal which

(A) has uncertainty (B) has no uncertainty

(C) is a completely specified function of time (D) none of the above

MCQ 1.1.6 Speech signals and the sine wave respectively are the example of

(A) deterministic signal, random signal.

(B) both random signals

(C) both deterministic signals

(D) random signal, deterministic signals

MCQ 1.1.7 Which of the following is a periodic signal ?

(A) $x(t) = At^2$ (B) $x(t) = Ae^{-j\alpha t}$

(C) $x(t) = Ae^{\alpha t}$ (D) $x(t) = Au(t)$

MCQ 1.1.8 The sum of two periodic signals having periods T_1 and T_2 is periodic only if the ratio of their respective periods (T_1/T_2) is

(A) an irrational number (B) a rational number

(C) an odd number (D) an even number

MCQ 1.1.9 A continuous-time signal $x(t)$ is said to be periodic with a fundamental period T_0, where T_0 is the

(A) smallest positive integer satisfying the relation $x(t) = x(t + mT_0)$ for any t and any m.

(B) positive constant satisfying the relation $x(t) = x(t + mT_0)$ for every t and any integer m.

(C) largest positive constant satisfying the relation $x(t) = x(t + mT_0)$ for any t and any integer m

(D) smallest positive constant satisfying the relation $x(t) = x(t + mT_0)$ for every t and any integer m

MCQ 1.1.10 Sine waves, cosine waves, square waves and triangular waves are the examples of

(A) non-deterministic functions (B) multiple frequency functions

(C) periodic functions (D) all of the above

MCQ 1.1.11 A signal is given by $x(t) = 2\cos(\omega t)\sin^2(\omega t) + 2\cos(\omega t) + \sin\omega(t) + \sin^2(\omega t)$. The odd component of $x(t)$ is

(A) $\cos(\omega t)\sin^2(\omega t)$ (B) $\sin(\omega t)$

(C) $\sin^2(\omega t)$ (D) $\cos(\omega t)$

MCQ 1.1.12 $f(t)$ is even while $g(t)$ is odd. If $x(t) = f(t) + g(t)$ and $y(t) = f(t)\,g(t)$ then $x(t)$ and $y(t)$ are respectively

(A) neither, even (B) odd, even

(C) neither, odd (D) even, odd

MCQ 1.1.13 Signal $x(t) = 5\sin 20\pi t$

(A) is an even signal (B) is an odd signal

(C) has even and odd parts (D) none of the above

MCQ 1.1.14 Which of the following statements is not true ?

1. The product of two even signals in an even signal
2. The product of two odd signals in an odd signal.
3. The product of even and odd signals in an even signal.
4. The product of even and odd signal is an odd signal.

(A) 2 and 3 (B) 1 only

(C) 3 only (D) 4 only

MCQ 1.1.15 $x(t) = 5\sin(10\pi t + 30^\circ)$

(A) is an odd signal

(B) is an even signal

(C) has an even part as well as an odd part

(D) none of the above

MCQ 1.1.16 The signal $x(t) = 10e^{j10\pi t}$ is

(A) an energy signal

(B) a power signal

(C) neither energy nor power signal

(D) both energy and power signal

MCQ 1.1.17 Signal $e^{-2t}u(t)$ is

(A) a power signal

(B) an energy signal

(C) neither an energy signal nor a power signal

(D) none of the above

MCQ 1.1.18 A signal is an energy signal if it has

(A) infinite energy

(B) finite energy

(C) zero average power

(D) both (B) and (C)

MCQ 1.1.19 A signal is a power signal if it has

(A) infinite energy

(B) infinite power

(C) finite power

(D) both (A) and (C)

MCQ 1.1.20 The signal $A\cos(\omega_0 t + \phi)$ is

(A) a periodic signal

(B) a power signal

(C) both periodic and power signals

(D) a energy signal

MCQ 1.1.21 Which of the following is an energy signal ?

(A) $x(t) = A\cos\omega_0 t$

(B) $x(t) = A\sin\omega_0 t$

(C) $x(t) = Ae^{j\omega_0 t}$

(D) $x(t) = e^{-at}u(t)$

MCQ 1.1.22 Which of the following statement are true ?

1. Most of the periodic signals are energy signals.
2. Most of the periodic signals are power signals.
3. For energy signals, the power is zero.
4. For power signals, the energy is zero.

(A) 1, 2 and 3 only

(B) 1 only

(C) 1 and 2 only

(D) 1, 2, 3, and 4

MCQ 1.1.23 A complex valued signal $x(t) = x_R(t) + jx_I(t)$ has conjugate symmetry if

(A) $x_R(t)$ is odd while $x_I(t)$ is even
(B) $x_R(t)$ and $x_I(t)$ are both odd
(C) $x_R(t)$ is even while $x_I(t)$ is odd
(D) $x_R(t)$ and $x_I(t)$ are both even

MCQ 1.1.24 A signal $x(t)$ has energy E_x, then energy of the signal $x(at)$ is given by

(A) $E_x/|a|^2$
(B) $E_x/|a|$
(C) $E_x|a|^2$
(D) $|a|E_x$

MCQ 1.1.25 The value of $\int_{-\pi}^{\pi} 2\cos\omega t\,\delta(\omega)\,d\omega$ is

(A) 0
(B) $\pi/2$
(C) 1
(D) 2

MCQ 1.1.26 If $\delta(t)$ is the unit impulse function, then $\int_{-\infty}^{\infty} x(t)\delta(t)\,dt$ equals to

(A) $x(t)$
(B) $x(0)$
(C) $x(\infty)$
(D) $x(1)$

MCQ 1.1.27 For unit impulse function $\delta(t)$, which of the following relation holds true ?

(A) $\delta(-t) = \delta\left(\frac{t}{2}\right)$
(B) $\delta(-t) = \delta(t^2)$
(C) $\delta(-t) = \delta(t)$
(D) $\delta(-t) = \delta^2(t)$

MCQ 1.1.28 The function $f(t) = t\delta(t)$ will be equal to

(A) t
(B) ∞
(C) 1
(D) 0

MCQ 1.1.29 The unit impulse is defined as,

(A) $\delta(t) = \infty,\ t = 0$
(B) $\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$
(C) $\delta(t) = \infty,\ t = 0$ and $\int_{-\infty}^{+\infty} \delta(t)\,dt = 0$
(D) $\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$ and $\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$

MCQ 1.1.30 If $x(t)$ is a continuous time signal and $\delta(t)$ is a unit impulse signal then value of integral $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)$ is equal to

(A) $x(t)$
(B) $x(t_0)$
(C) $\delta(t)$
(D) 1

MCQ 1.1.31 A weighted impulse function $\delta(at)$ has

(A) unit area and unit amplitude
(B) infinite area and finite amplitude
(C) finite area and infinite amplitude
(D) infinite area and infinite amplitude

MCQ 1.1.32 Unit step signal $u(t)$ is

(A) an energy signal

(B) a power signal

(C) neither power signal nor energy signal

(D) both

MCQ 1.1.33 A unit step function is given by

(A) $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

(B) $u(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$

(C) $u(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

(D) $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$

MCQ 1.1.34 Match List I with List II and choose the correct answer using the codes given below the lists :

	List I (Signal)		**List II (Nature)**
P.	Unit Impulse signal	**1.**	Sample values are unpredictable
Q.	Unit Step signal	**2.**	Has only one non-zero value
R.	Random noise signal	**3.**	Amplitude decreases as time increases
S.	Decaying exponential	**4.**	Has only two possible values

Codes :

	P	Q	R	S
(A)	3	2	4	1
(B)	2	4	1	3
(C)	1	2	3	4
(D)	2	1	4	3

MCQ 1.1.35 A unit ramp function is defined as

(A) $r(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$

(B) $r(t) = \begin{cases} \vert t \vert + 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

(C) $r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

(D) $r(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

MCQ 1.1.36 The differentiation of a unit step signal is,

(A) an impulse signal

(B) a ramp signal

(C) an exponential signal

(D) a parabolic signal

MCQ 1.1.37 In terms of unit-step function, signum function is given as

(A) $\text{sgn}(t) = - u(t)$

(B) $\text{sgn}(t) = 2u(t)$

(C) $2\text{sgn}(t) = u(t)$

(D) $\text{sgn}(t) = 2u(t) - 1$

MCQ 1.1.38 The signum function is defined as

(A) $\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$

(B) $\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$

(C) $\text{sgn}(t) = \begin{cases} 0, & t > 0 \\ -1, & t < 0 \end{cases}$

(D) $\text{sgn}(t) = \begin{cases} -1, & t > 0 \\ 1, & t < 0 \end{cases}$

MCQ 1.1.39 Differentiation of signum function will be

(A) $\frac{1}{2}\delta(t)$

(B) $\delta(t)$

(C) $2\delta(t)$

(D) $2u(t)$

MCQ 1.1.40 The sinc function $f(t)$ is defined as

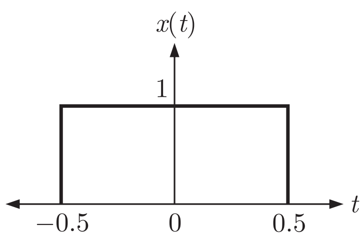
(A) $f(t) = \frac{\sin \pi t}{\pi t}$

(B) $f(t) = \frac{\sin t}{\pi t}$

(C) $f(t) = \frac{\sin \pi t}{t}$

(D) $f(t) = \frac{\sin \pi t}{t}$

MCQ 1.1.41 The mathematical expression for the signal $x(t)$ shown in figure is given by



(A) $u(t - 0.5) + u(t + 0.5)$

(B) $u(t + 0.5) - u(t - 0.5)$

(C) $u(t - 0.5) - u(t - 0.5)$

(D) $u(t + 0.5) + u(t - 0.5)$

****

EXCERCISE 1.2

MCQ 1.2.1 What is the period of a signal $x(t) = 3\sin(4\pi t) + 7\cos(3\pi t)$?

(A) 2 sec
(B) 4 sec
(C) 12 sec
(D) $x(t)$ is not periodic

MCQ 1.2.2 The period of a signal $x(t) = 3\sin(4\pi t) + 7\cos(10t)$ is

(A) 10π sec
(B) 5 sec
(C) 6 sec
(D) $x(t)$ is not periodic

MCQ 1.2.3 Consider the following continuous time signals

$$x_1(t) = 6\sin(8\pi t) + 14\cos(6\pi t)$$
$$x_2(t) = 6\sin(8\pi t) + 14\cos(20t)$$

Which of the following statement regarding the periodicity of the signals is true ?

(A) $x_1(t)$ is periodic, $x_2(t)$ is aperiodic
(B) Both $x_1(t)$ and $x_2(t)$ are periodic
(C) $x_1(t)$ is aperiodic, $x_2(t)$ is periodic
(D) Both $x_1(t)$ and $x_2(t)$ are aperiodic

MCQ 1.2.4 What is the period of the signal $x(t) = \sin\left(\frac{2\pi}{5}t\right)\cos\left(\frac{4\pi}{3}t\right)$?

(A) 13 sec
(B) 91 sec
(C) 15 sec
(D) $x(t)$ is aperiodic

MCQ 1.2.5 Match List I (Signal) with List II (Period of the signal) and select the answer using the codes given below

	List I (Signals)		**List II (Period of the signal)**
P.	$f_1(t) = \sin\left(\frac{2\pi}{3}\right)t$	**1.**	15 Unit
Q.	$f_2(t) = \sin\left(\frac{2\pi}{5}t\right)\cos\left(\frac{4\pi}{3}t\right)$	**2.**	3 Unit
R.	$f_3(t) = \sin 3t$	**3.**	aperiodic
S.	$f_4(t) = f_1(t) - 2f_3(t)$	**4.**	$2\pi/3$ unit

Codes :

	P	Q	R	S
(A)	1	4	3	2
(B)	3	2	1	4
(C)	1	2	3	4
(D)	2	1	4	3

MCQ 1.2.6 Which of the following signal is not periodic?

(A) $\sin(10t)$

(B) $2\cos(5\pi t)$

(C) $\sin(10\pi t)\,u(t)$

(D) none of these

MCQ 1.2.7 The period of the signal $g(t) = 2\cos(10t+1) + \sin(4t-1)$ is equal to

(A) 10 sec

(B) π sec

(C) 2 sec

(D) 5 sec

MCQ 1.2.8 Consider the signals $x_1(t) = 5\cos(4t+\frac{\pi}{3})$, $x_2(t) = e^{j(\pi t-1)}$ and $x_3(t) = \left[\cos(2t-\frac{\pi}{3})\right]^2$ Which signals is/are aperiodic

(A) $x_3(t)$ only

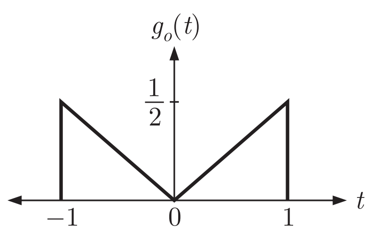
(B) $x_2(t)$ and $x_3(t)$

(C) $x_2(t)$ only

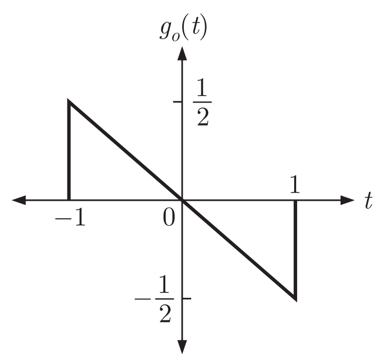
(D) none of above

MCQ 1.2.9 Consider a signal $g(t)$ defined as $g(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$. The odd part of $g(t)$ is

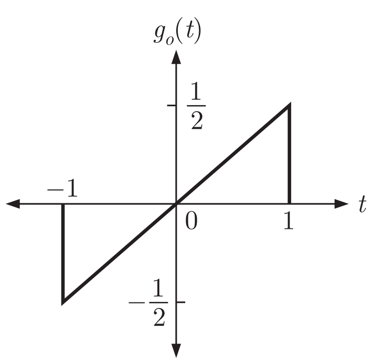
(A)



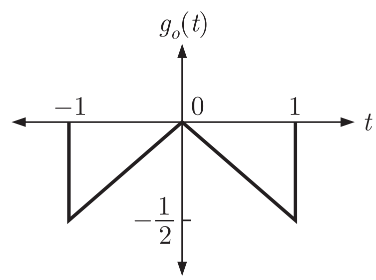
(B)



(C)



(D)



MCQ 1.2.10 A signal $g(t)$ is defined as

$$g(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}.$$

The even part of the signal $g(t)$ is

(A) $g_e(t) = \begin{cases} t/2, & -1 \le t < 0 \\ t/2, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$

(B) $g_e(t) = \begin{cases} -t/2, & -1 \le t < 0 \\ t/2, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$

(C) $g_e(t) = \begin{cases} -2t, & -1 \le t < 0 \\ 2t, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$

(D) $g_e(t) = \begin{cases} 2t, & -1 \le t < 0 \\ 2t, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$

MCQ 1.2.11 A CT signal is defined as

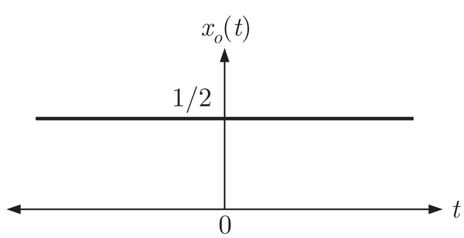
$$x(t) = \begin{cases} 2, & t > 0 \\ 0, & t < 0 \end{cases}$$

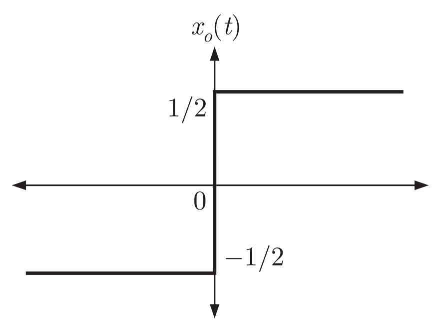
The odd part of $x(t)$ is an unit

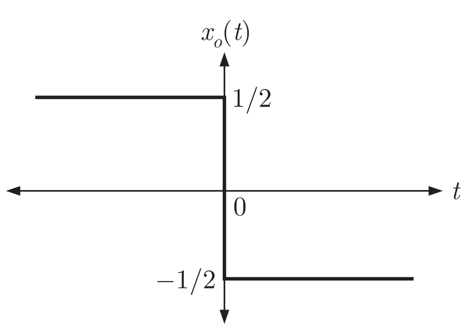
(A) step function (B) signum function

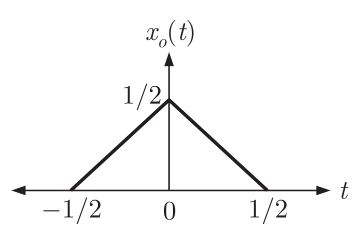
(C) impulse function (D) ramp function

MCQ 1.2.12 The odd part of a unit step signal is

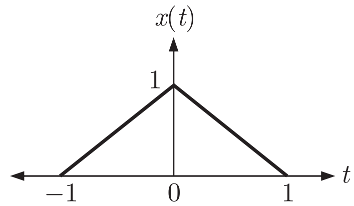
(A)


(B)


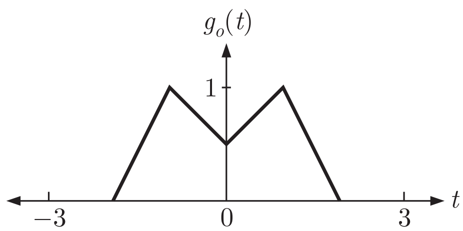
(C)


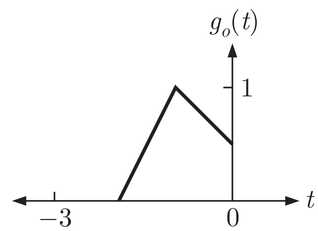
(D)


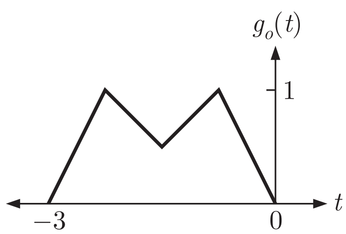
MCQ 1.2.13 A signal $x(t)$ is shown in figure below



The odd part of the signal $g(t) = x(t - \frac{3}{4}) + x(t + \frac{3}{4})$ will be

(A) 

(B) 

(C) 

(D) None of above

MCQ 1.2.14 If $x_e(t)$ and $x_o(t)$ are the even and odd part of a signal $x(t)$, then which of the following is true?

(A) $x_o(0) = 0$

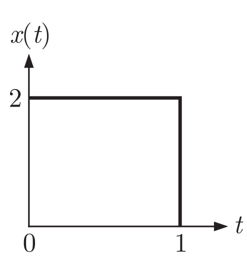
(B) $x_e(0) = x(0)$

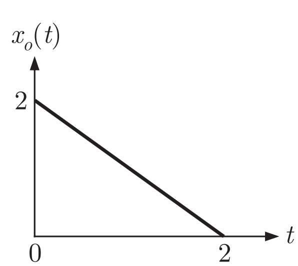
(C) $x_o(0) = x_e(0) = 0$

(D) Both (A) and (B)

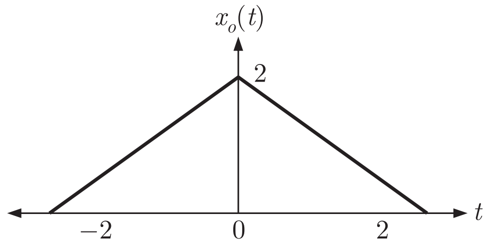
Statement For Q. 15 & 16 :

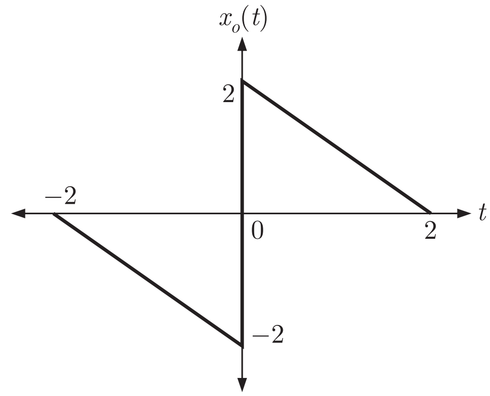
The figure shows parts of a signal $x(t)$ and its odd part $x_o(t)$, for $t \geq 0$ only, that is $x(t)$ and $x_o(t)$ are not given for $t < 0$.

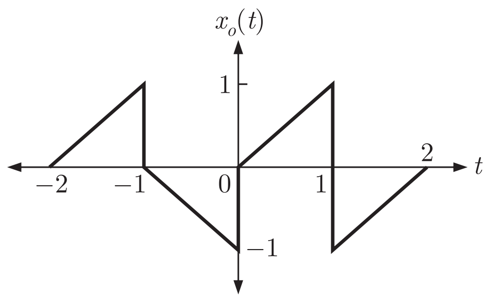




MCQ 1.2.15 The complete odd part $x_o(t)$ of the signal will be

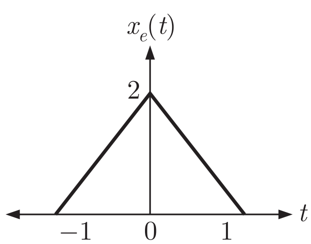
(A)


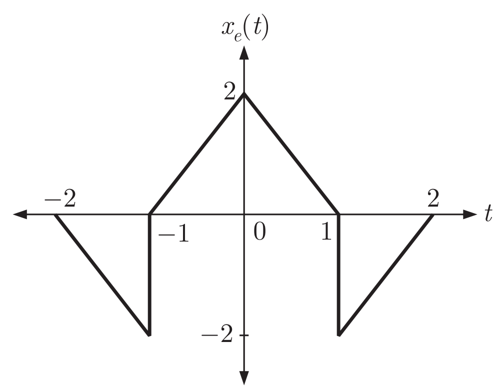
(B)


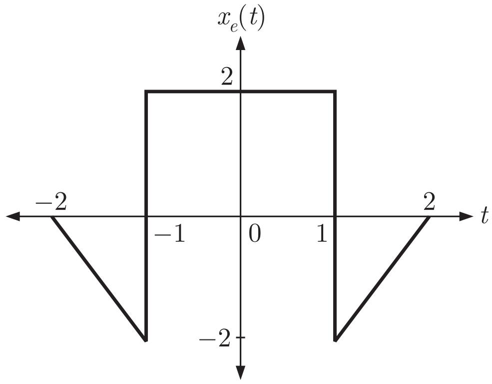
(C)


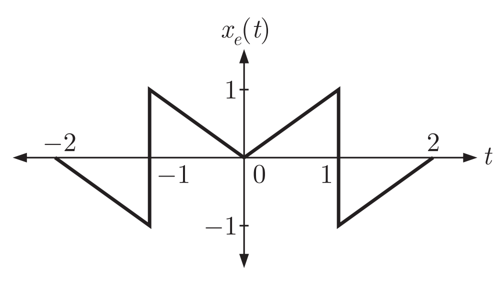
(D) Cannot be determined

MCQ 1.2.16 The complete even part $x_e(t)$ of the signal $x(t)$ is

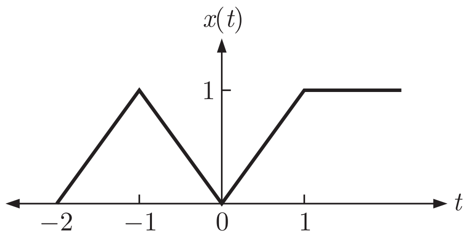
(A)


(B)


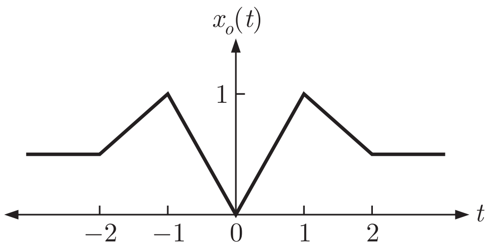
(C)


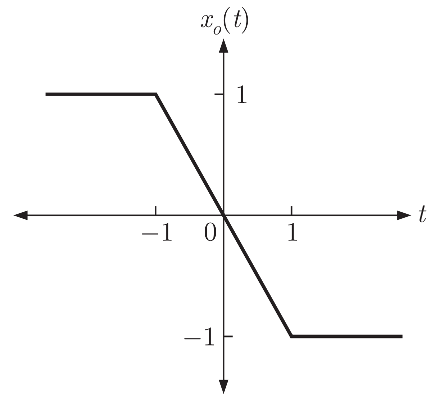
(D)


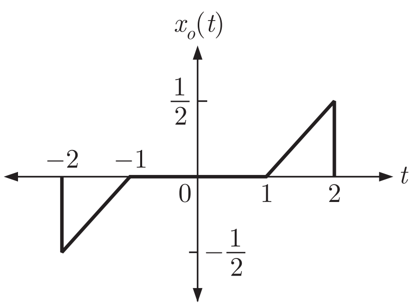
MCQ 1.2.17 A signal $x(t)$ is shown in figure below

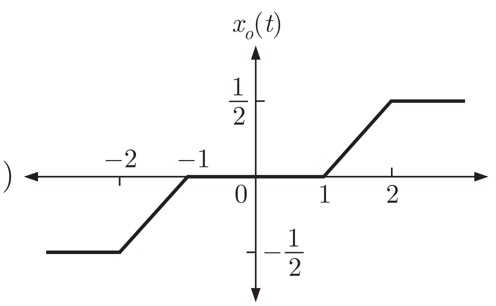


The odd part of signal $x(t)$ is

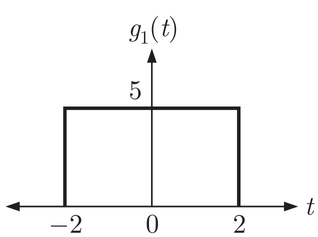
(A)


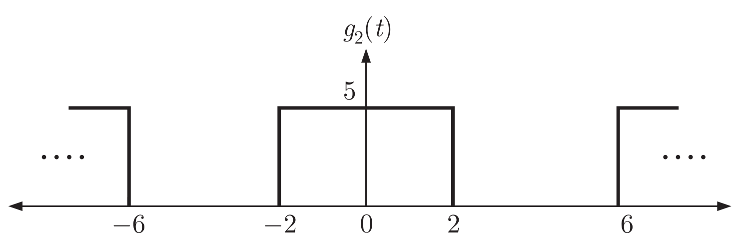
(B)


(C)


(D)


MCQ 1.2.18 Two signals $g_1(t)$ and $g_2(t)$ are shown in the following figures





Which of the following statement is true ?

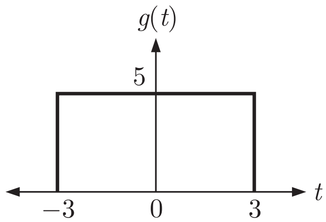
(A) $g_1(t)$ is a power signal, $g_2(t)$ is an energy signal.

(B) $g_1(t)$ is an energy signal, $g_2(t)$ is a power signal.

(C) Both $g_1(t)$ and $g_2(t)$ are power signals.

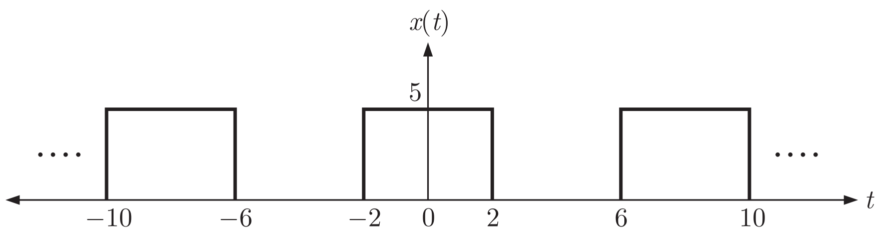
(D) Both $g_1(t)$ and $g_2(t)$ are energy signals.

MCQ 1.2.19 The average power (P_g) and energy (E_g) of the signal $g(t)$ shown in figure are



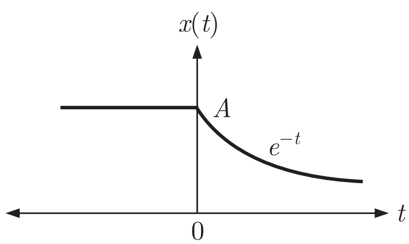
(A) $P_g = 25$, $E_g = 150$
(B) $P_g = 0$, $E_g = 150$
(C) $P_g = 25$, $E_g = \infty$
(D) $P_g = 25$, $E_g = 50$

MCQ 1.2.20 The energy and average power of a signal $x(t)$ as shown in figure are respectively :



(A) 100, 0
(B) ∞, 25
(C) 50, 0
(D) ∞, 12.5

MCQ 1.2.21 The energy of the signal shown in figure is



(A) $A^2/2$
(B) A^2
(C) $A^2/4$
(D) None of above

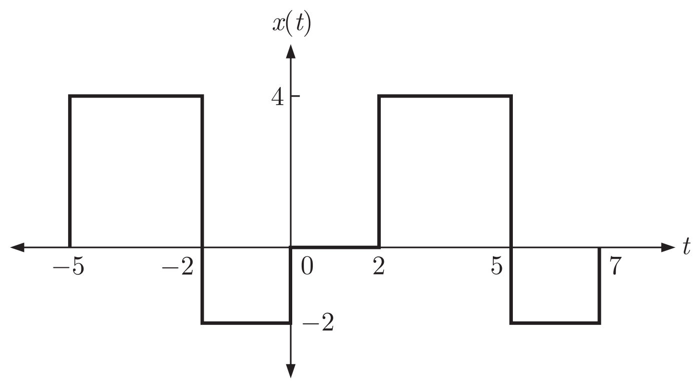
MCQ 1.2.22 The power and rms value of a voltage signal $x(t) = 20\cos(5t)\cos(10t)\,\text{V}$ are respectively :

(A) 200 W, 14.14 volt
(B) 100 W, 7.07 volt
(C) 100 W, 10 volt
(D) 200 W, 10 volt

MCQ 1.2.23 The signal $x(t) = e^{j\left(2t + \frac{\pi}{4}\right)}$ is

(A) a power signal
(B) an energy signal
(C) neither a power nor an energy
(D) none of above

MCQ 1.2.24 The power of a periodic signal shown in figure is



(A) 56 unit (B) 8 unit

(C) 11.2 unit (D) 32 unit

MCQ 1.2.25 A signal $x(t)$, defined over the range $-3 \le t \le 3$, has energy equal to 12 units. Match List I (signal) with List II (Energy of the signal) and select correct answer using the codes given below

	List I (Signal)		List II (Energy)
P.	$2x(t)$	**1.**	48 unit
Q.	$x(3t)$	**2.**	12 unit
R.	$x(t-4)$	**3.**	4 unit
S.	$2x(2t)$	**4.**	24 unit

Codes:

	P	Q	R	S
(A)	1	3	2	4
(B)	4	3	1	2
(C)	1	4	3	2
(D)	4	1	2	3

MCQ 1.2.26 Consider the following statements regarding a signal $x(t) = e^{-|t|}$.

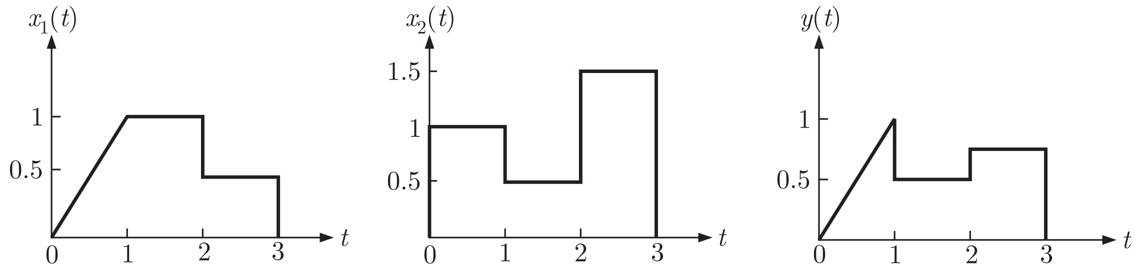
1. $x(t)$ is an energy signal
2. $x(t)$ is an odd signal
3. $x(t)$ is an even signal
4. $x(t)$ is neither even nor odd.

Which of the above statement is/are true?

(A) only 4 (B) 1 and 3

(C) 1 and 4 (D) 1 and 2

MCQ 1.2.27 Consider the signals $x_1(t)$, $x_2(t)$ and $y(t)$ as shown in below :



Which of the following relation is true ?

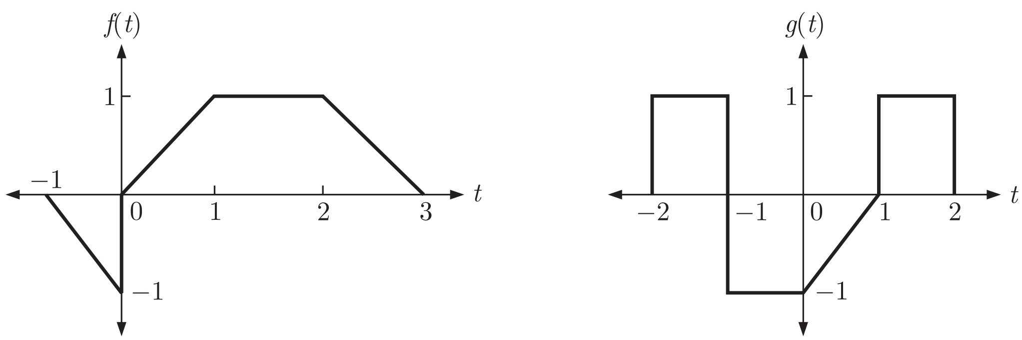
(A) $y(t) = x_1(t)\, x_2(t)$

(B) $y(t) = x_1(t) + x_2(t)$

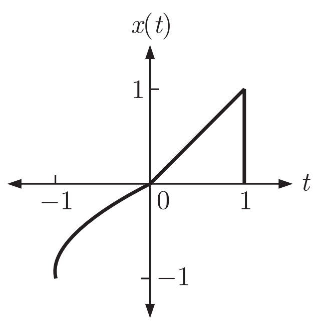
(C) $y(t) = x_1(t) - x_2(t)$

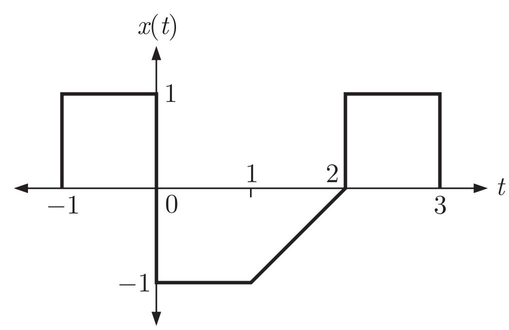
(D) none of above

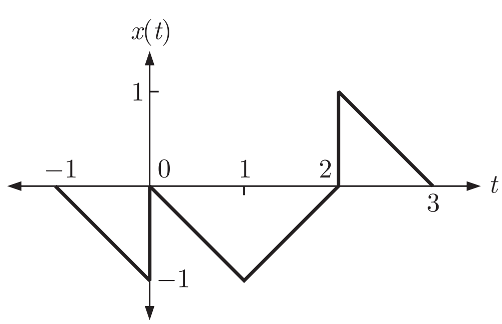
MCQ 1.2.28 Two CT signals $f(t)$ and $g(t)$ are shown in following figure :

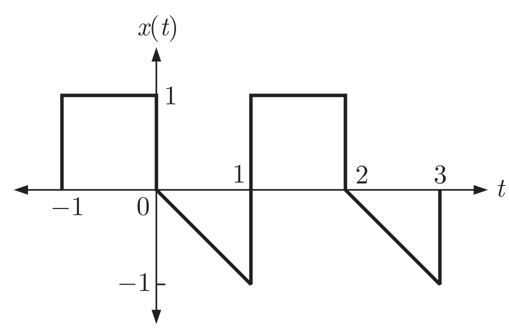


The plot for a signal $x(t) = f(t)\, g(t-1)$ will be

(A)


(B)


(C)


(D)


MCQ 1.2.29 A continuous time signal is given as

$$g(t) = \begin{cases} t+1, & -1 \le t \le 0 \\ 1, & 0 \le t < 2 \\ 0, & \text{elsewhere} \end{cases}$$

The correct expression for $g(2t)$ is

(A) $g(2t) = \begin{cases} \frac{t}{2}+1, & -0.5 \le t \le 0 \\ t, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$

(B) $g(2t) = \begin{cases} 2t+1, & -0.5 \le t \le 0 \\ 2, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$

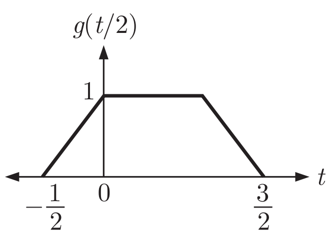
(C) $g(2t) = \begin{cases} t+1, & -0.5 \le t \le 0 \\ 1, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$

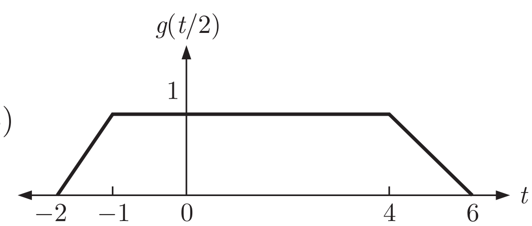
(D) $g(2t) = \begin{cases} 2t+1, & -0.5 \le t \le 0 \\ 1, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$

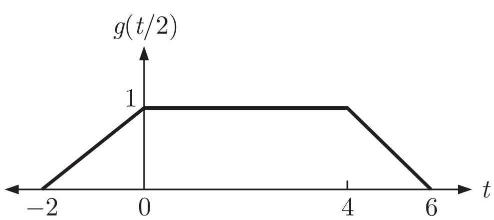
MCQ 1.2.30 Consider a signal $g(t)$ defined as following

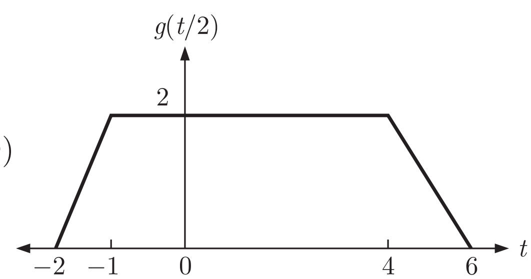
$$g(t) = \begin{cases} t+1, & -1 \le t \le 0 \\ 1, & 0 \le t \le 2 \\ -t+3, & 2 \le t \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

The waveform of signal $g(t/2)$ is

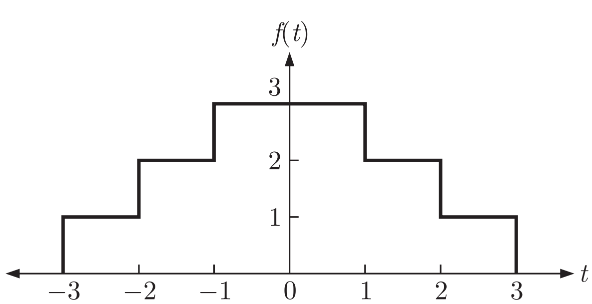
(A)


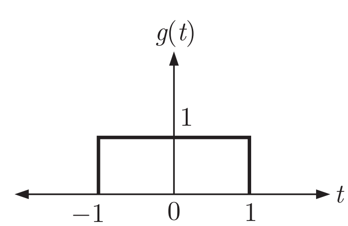
(B)


(C)


(D)


MCQ 1.2.31 Two signals $f(t)$ and $g(t)$ are shown in the figure below





Which of the following is the correct expression of $f(t)$?

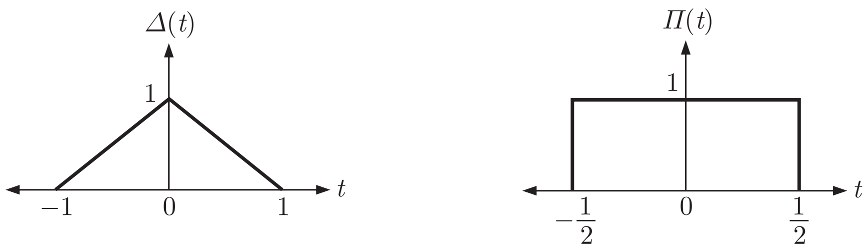
(A) $f(t) = g(t) + g(t+2) + g(t+3)$

(B) $f(t) = g(t) + g(t-2) + g(t-3)$

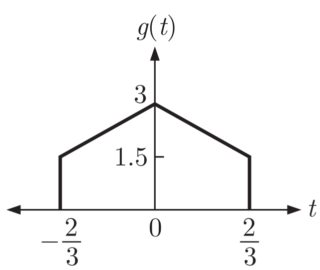
(C) $f(t) = g(t) + g(t/2) + g(t/3)$

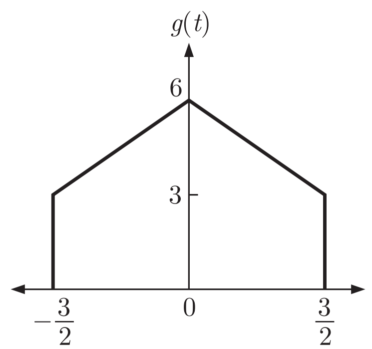
(D) $f(t) = g(t) + g(2t) + g(3t)$

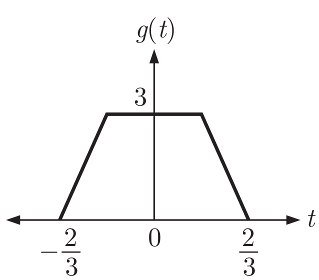
MCQ 1.2.32 Consider a unit triangular function $\Delta(t)$ and a unit rectangular function $\Pi(t)$ as shown in figure

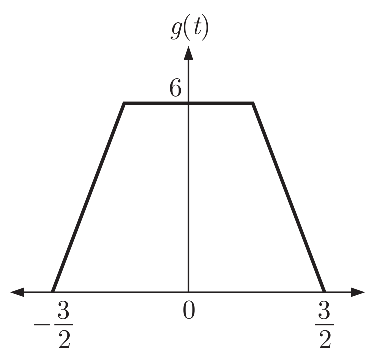


Which of the following waveform is correct for $g(t) = 3\Delta(2t/3) + 3\Pi(t/3)$

(A)


(B)


(C)


(D)


MCQ 1.2.33 Time compression of a signal

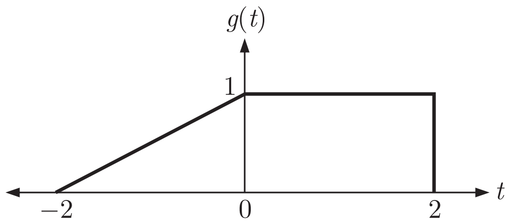
(A) Reduces its energy

(B) increases its energy

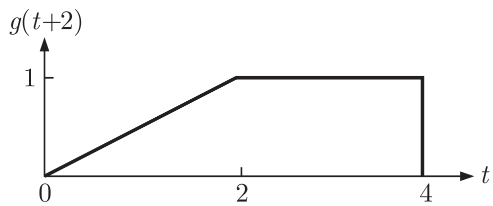
(C) does not effect the energy

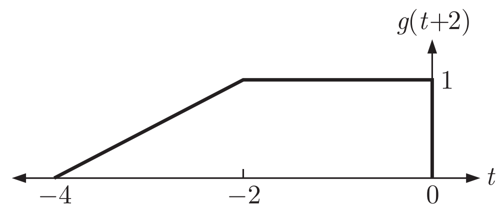
(D) none of above.

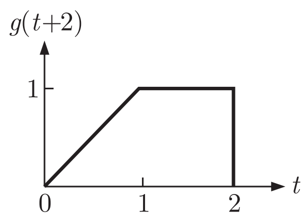
MCQ 1.2.34 A CT signal is shown below

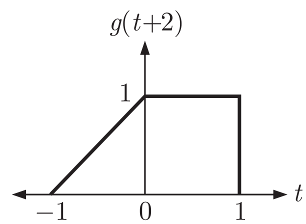


The plot of signal $g(t+2)$ is

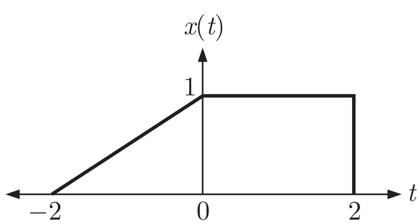
(A)


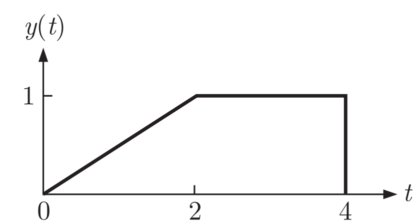
(B)


(C)


(D)


MCQ 1.2.35 Consider the signal $x(t)$ and $y(t)$ shown is figures





Which of the following is correct statement ?

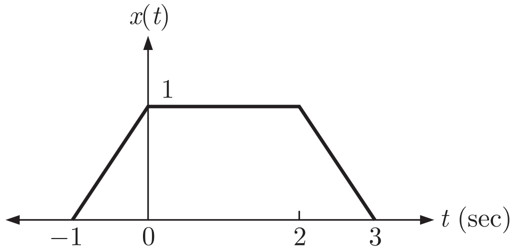
(A) $y(t)$ is amplitude scaled version of $x(t)$

(B) $y(t)$ is time scaled version of $x(t)$ by a factor of 2.

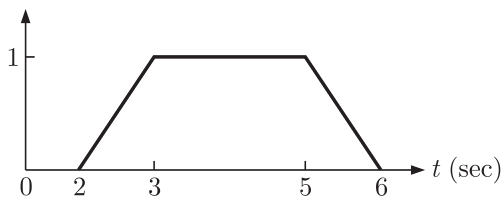
(C) $y(t)$ is time advanced version of $x(t)$ by 2 units.

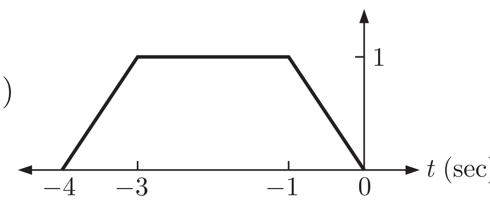
(D) $y(t)$ is time delayed version of $x(t)$ by 2 units.

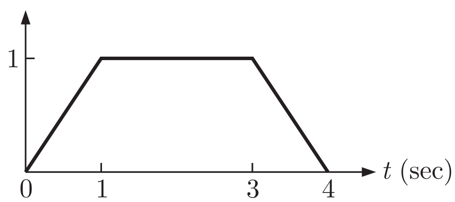
MCQ 1.2.36 The plot of a signal $x(t)$ is shown in figure

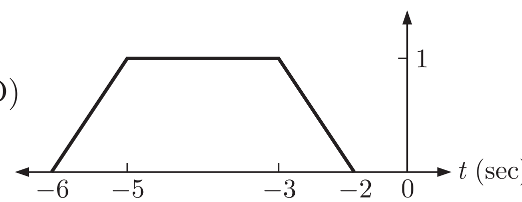


If $x(t)$ is delayed by 3 sec, then plot will be

(A)


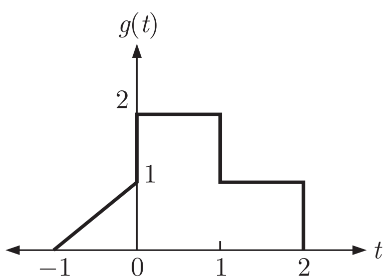
(B)


(C)


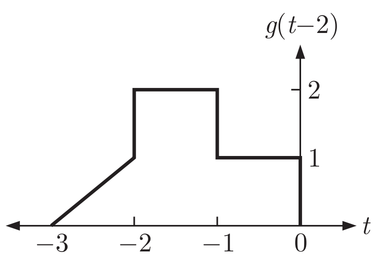
(D)


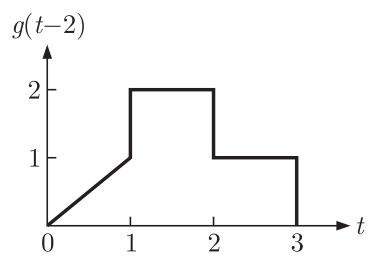
Statement For Q. 37 & 38

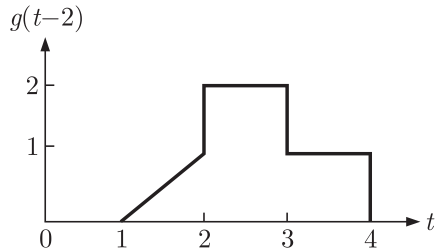
Consider the signal $g(t)$ as shown in figure

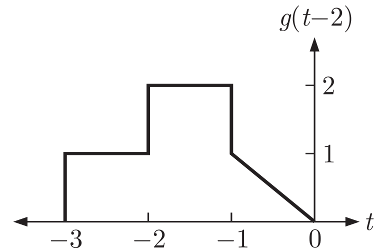


MCQ 1.2.37 Plot for signal $g(t-2)$ will be

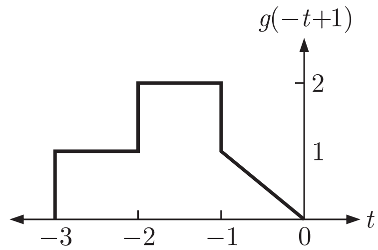
(A)


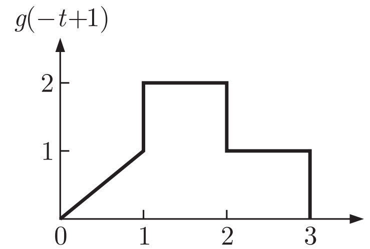
(B)


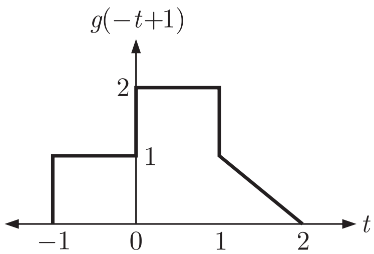
(C) 

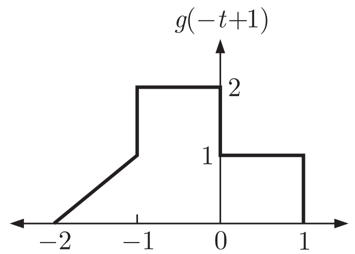
(D) 

MCQ 1.2.38 Plot for signal $g(-t+1)$ will be

(A) 

(B) 

(C) 

(D) 

MCQ 1.2.39 If the energy of a signal $x(t)$ is E_x then what will be the energy for a signal $x(at-b)$?

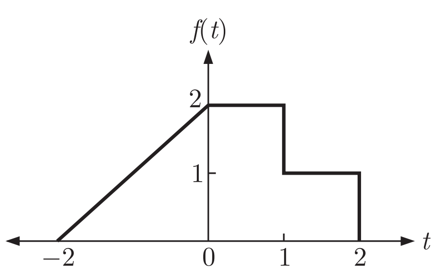
(A) $\frac{E_x}{a}$

(B) $\left(\frac{b}{a}\right)E_x$

(C) $\frac{1}{a}E_x+b$

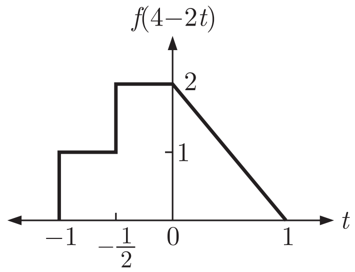
(D) $\left(\frac{1}{a}+b\right)E_x$

MCQ 1.2.40 Consider a signal $f(t)$ as shown is figure

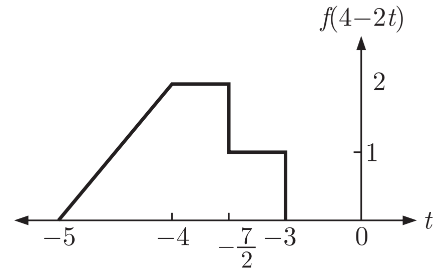


The plot of signal $f(4-2t)$ is

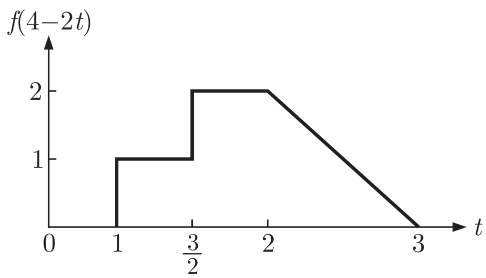
(A)



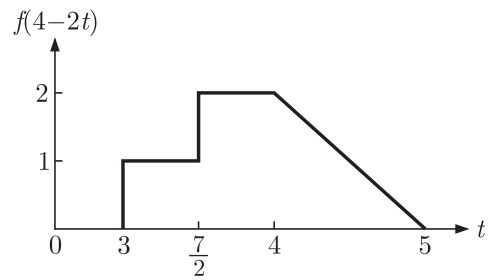
(B)



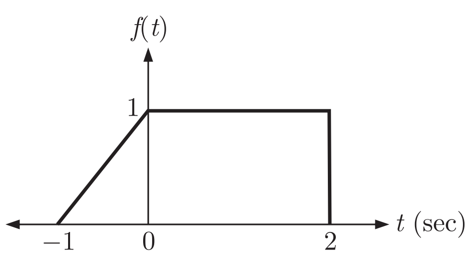
(C)



(D)

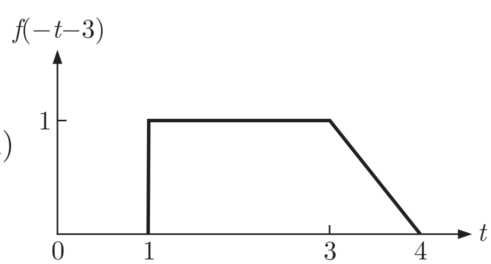


MCQ 1.2.41 If plot of a signal $f(t)$ is shown in figure below

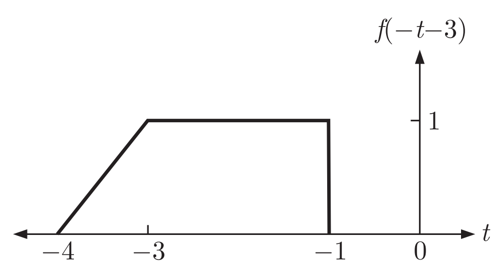


Then the plot of signal $f(-t-3)$ will be

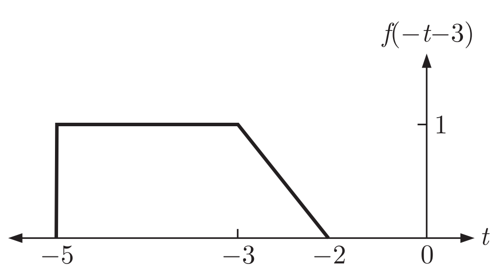
(A)



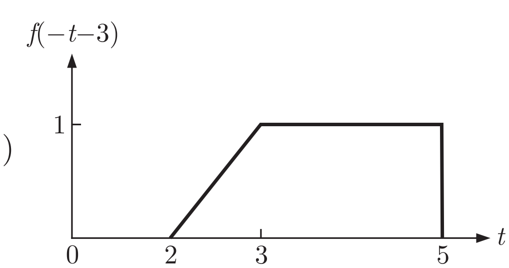
(B)



(C)



(D)



MCQ 1.2.42 A signal $x(t)$ and its transformed signal $y(t)$ are shown in figure(A) and figure(B) respectively

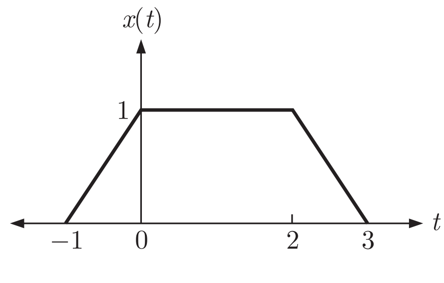


Fig (A)

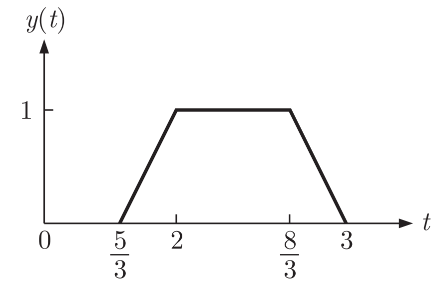


Fig (B)

If $y(t) = x(at + b)$, then values of a and b are respectively

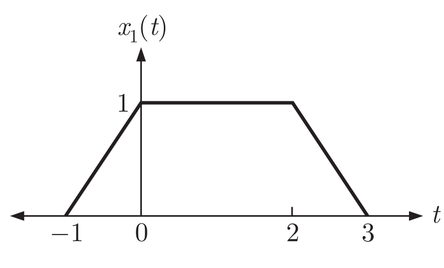
(A) 3, −2

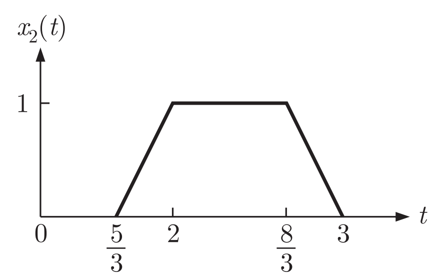
(B) −3, 6

(C) 3, −6

(D) −2, 3

MCQ 1.2.43 Consider two signals $x_1(t)$ and $x_2(t)$ as shown below





Which of the following procedure is correct to obtain $x_2(t)$ from $x_1(t)$?

(A) First compress $x_1(t)$ by a factor of 3, then shift to the right by 6 time units.

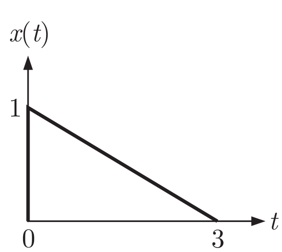
(B) First expand $x_1(t)$ by a factor of 6, then shift to the right by 3 time units.

(C) First compress $x_1(t)$ by a factor of 3, then shift to the right by 2 time units.

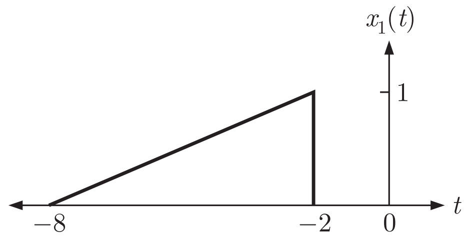
(D) First shift $x_1(t)$ to the right by 2 time units then expand by a factor of 3.

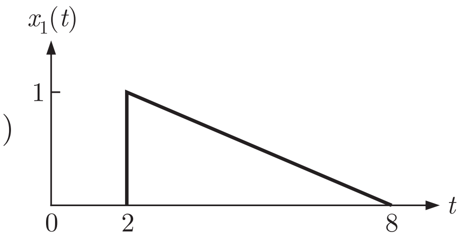
Statement For Q. 44 & 45

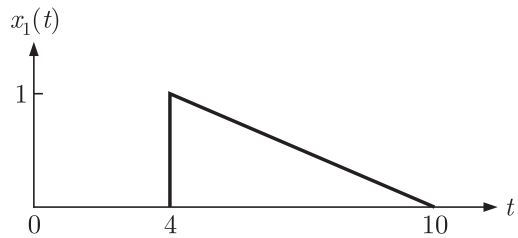
The plot of a signal $x(t)$ is shown in figure

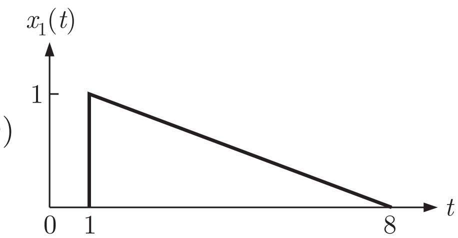


MCQ 1.2.44 Plot for the signal $x_1(t) = x[0.5(t-2)]$ will be

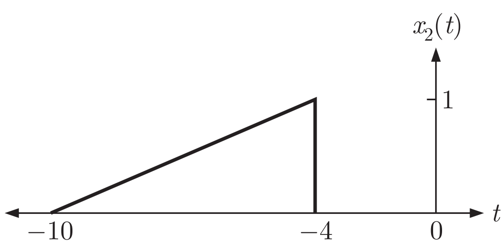
(A)


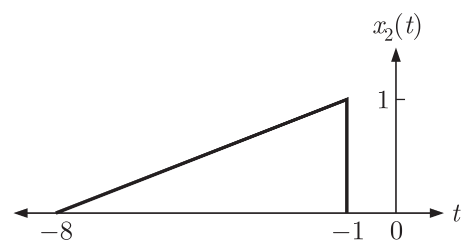
(B)


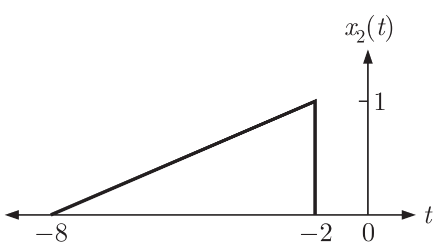
(C)


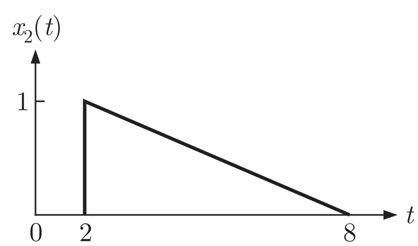
(D)


MCQ 1.2.45 Plot for the signal $x_2(t) = x(-0.5t-1)$ will be

(A)


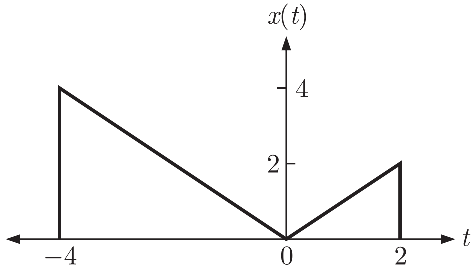
(B)


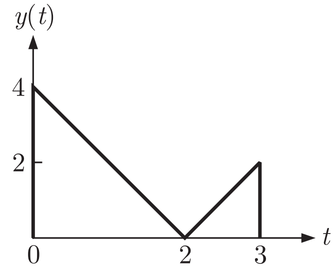
(C)


(D)


Statement For Q. 46 & 47

Consider two CT signal $x(t)$ and $y(t)$ shown in figure below



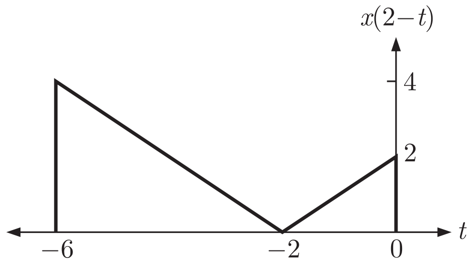


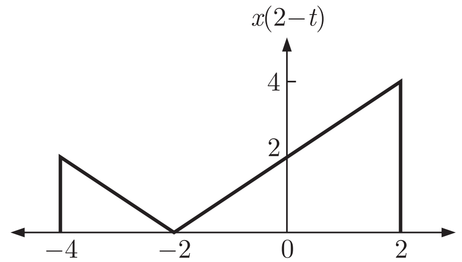
MCQ 1.2.46 Which of the following relation is true ?

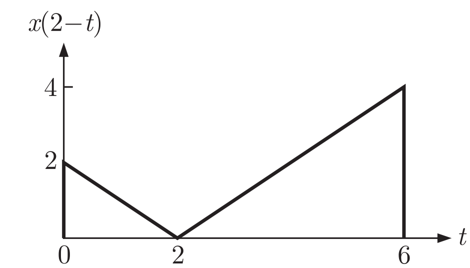
(A) $y(t) = x(2t-8)$ (B) $y(t) = x(2t-4)$

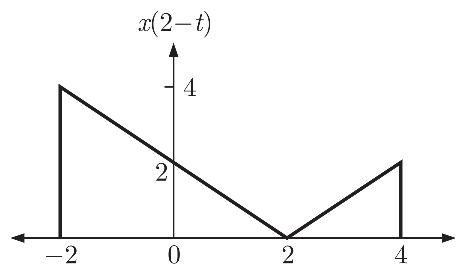
(C) $y(t) = x\left(\frac{t}{2}-2\right)$ (D) $y(t) = x\left(\frac{t}{2}-4\right)$

MCQ 1.2.47 The sketch of signal $x(2-t)$ will be

(A)


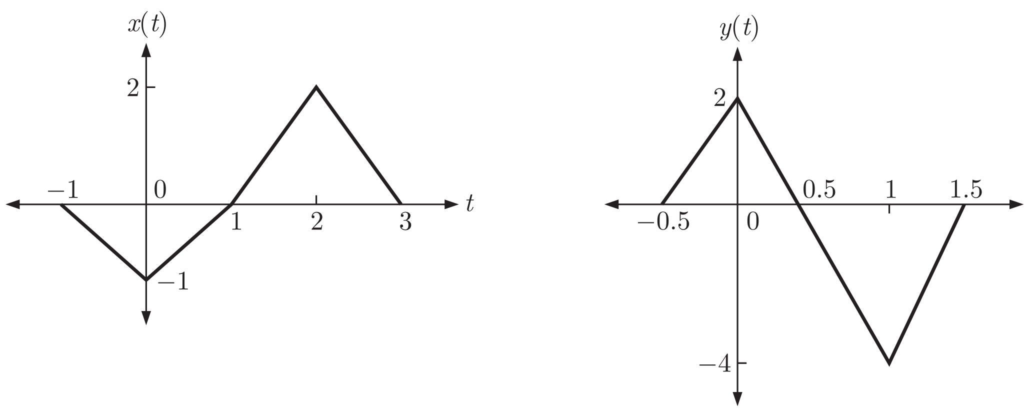
(B)


(C)


(D)


MCQ 1.2.48 Consider two signals $x(t)$ and $y(t)$ shown in figure below

1.4

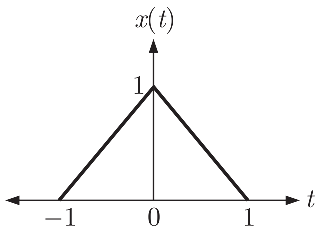


If $y(t) = Ax\left(\frac{t-t_0}{W}\right)$ then, the values of A, t_0 and W are respectively.

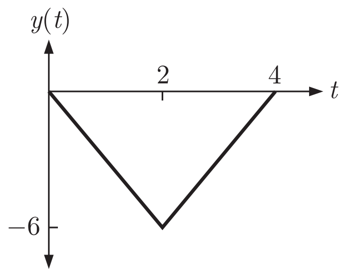
(A) $-2, 0, 2$ (B) $-2, 1, \frac{1}{2}$

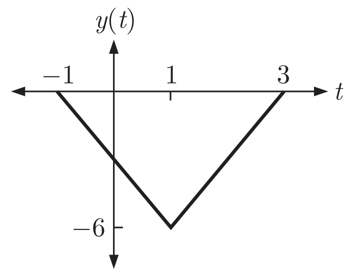
(C) $-2, 0, \frac{1}{2}$ (D) $2, 1, 2$

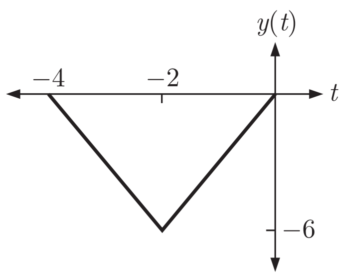
MCQ 1.2.49 A signal $x(t)$ is shown in the following figure



The plot for a transformed signal $y(t) = -6x\left(\frac{t-1}{2}\right)$ will be

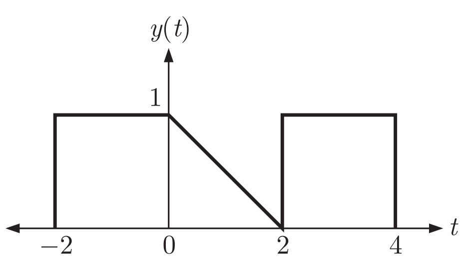
(A) 

(B) 

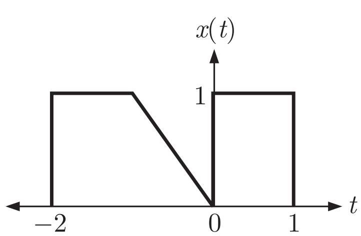
(C) 

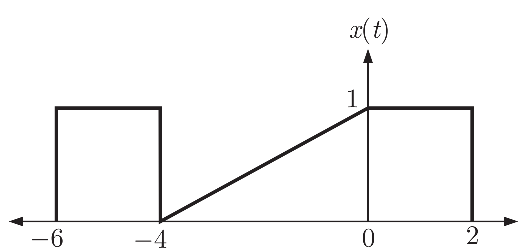
(D) None of above

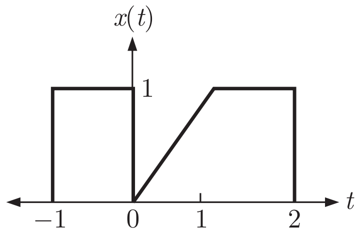
MCQ 1.2.50 A signal $x(t)$ is transformed into another signal $y(t)$ given as $y(t) = x\left(1 - \frac{t}{2}\right)$

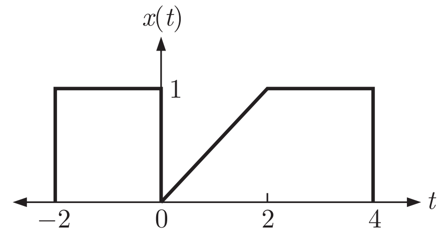


The waveform of the original signal $x(t)$ is

(A) 

(B) 

(C)


(D)


MCQ 1.2.51 If $\delta(t)$ is an unit impulse function, then the value of integral $\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)\,dt$ equals to

(A) $\frac{1}{2}$

(B) $\frac{1}{e}$

(C) $\frac{1}{2e}$

(D) 1

MCQ 1.2.52 For an unit impulse function $\delta(t)$, which of the following is true?

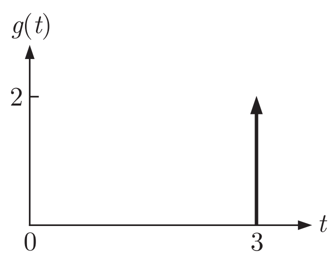
(A) $\delta[a(t-t_0)] = \frac{1}{|a|}\delta(t)$

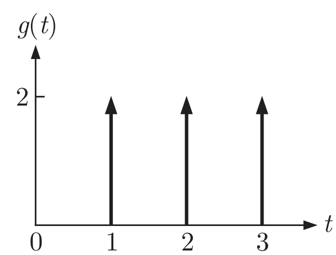
(B) $\delta[a(t-t_0)] = |a|\delta(t-t_0)$

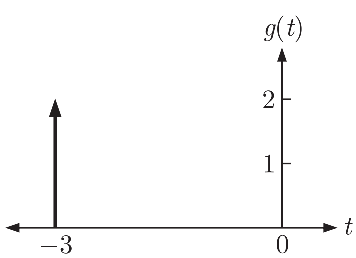
(C) $\delta[a(t-t_0)] = \frac{1}{|a|}\delta(t-t_0)$

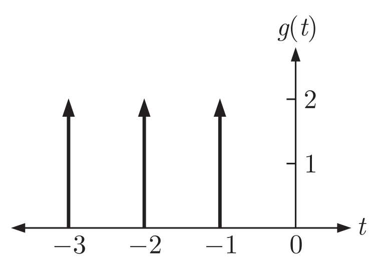
(D) $\delta[a(t-t_0] = |a|\delta(t)$

MCQ 1.2.53 If $\delta(t)$ is an unit impulse function then which of the following waveform represents a signal $g(t) = 6\delta(3t+9)$?

(A)


(B)


(C)


(D)


MCQ 1.2.54 What is the numerical value of the following integral

$$x(t) = \int_{-\infty}^{\infty} \delta(t+5)\cos(\pi t)\,dt$$

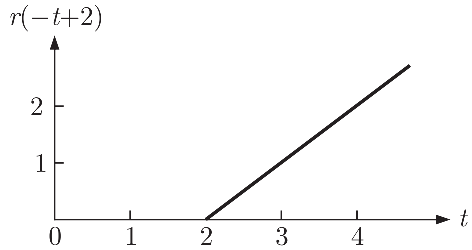
(A) 1

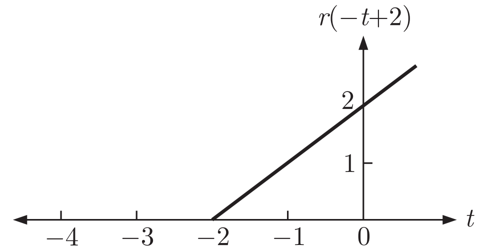
(B) -1

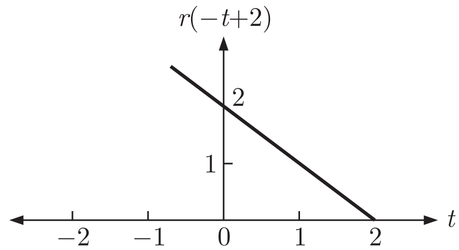
(C) 0

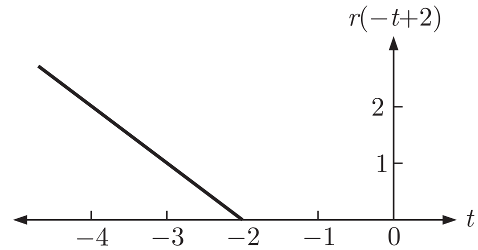
(D) 5

MCQ 1.2.55 If $r(t)$ is a unit ramp function, then plot for signal $r(-t+2)$ will be

(A)


(B)


(C)


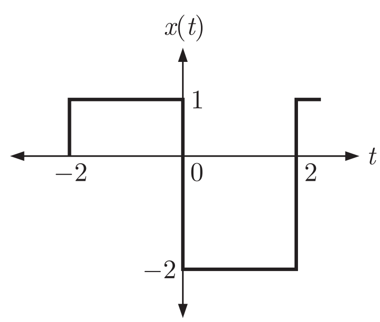
(D)


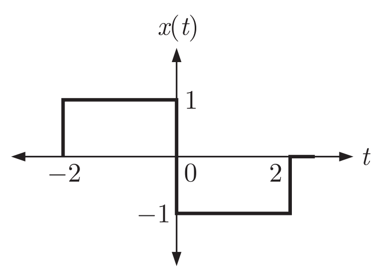
MCQ 1.2.56 Consider three signals $x_1(t) = u(t) - u(t-1)$, $x_2(t) = r(t) - r(t-2)$ and $x_3(t) = (1 + e^{-6t})\,u(t)$ where $u(t)$ and $r(t)$ are unit-step function and unit-ramp function respectively. Which of the above signals have finite energy?

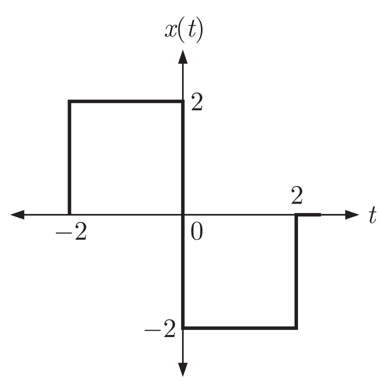
(A) $x_1(t)$ and $x_3(t)$ (B) $x_1(t)$ only

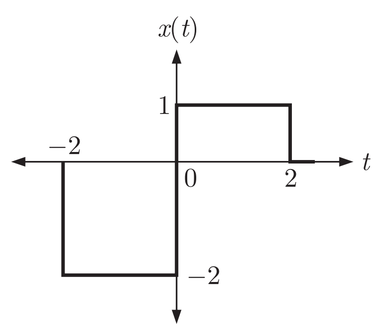
(C) $x_2(t)$ and $x_3(t)$ (D) $x_2(t)$ only

MCQ 1.2.57 For a signal $x(t) = u(t+2) - 2u(t) + u(t-2)$ the waveform is

(A)


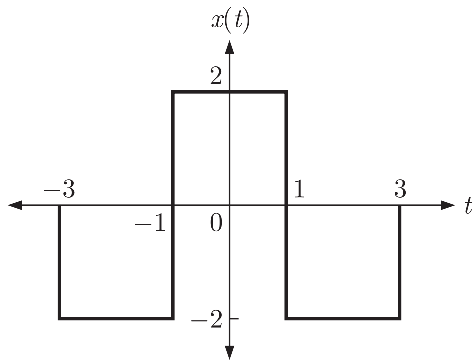
(B)


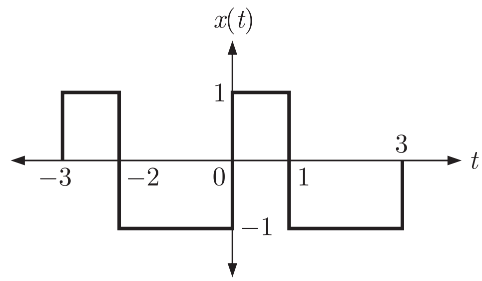
(C)


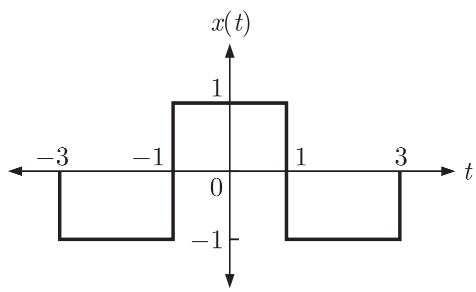
(D)


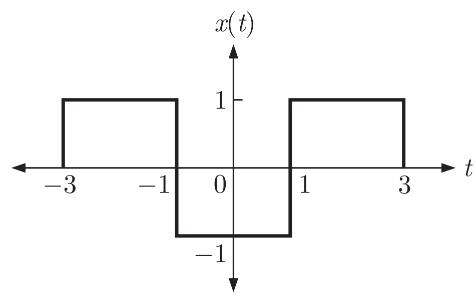
MCQ 1.2.58 Which of the following is correct waveform of a signal $x(t)$ given as below

$$x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$$

(A) 

(B) 

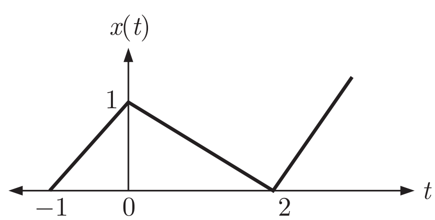
(C) 

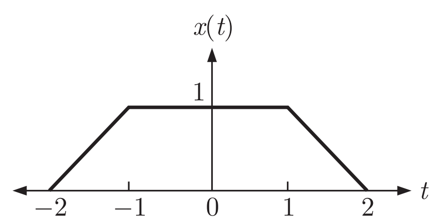
(D) 

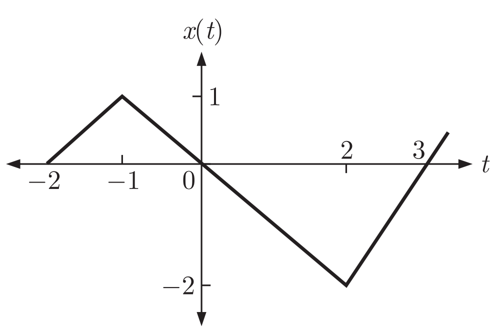
MCQ 1.2.59 Consider a signal $x(t)$ which is a linear combination of ramp signals given as

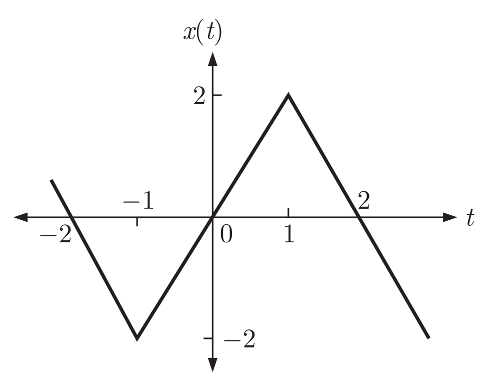
$$x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$

The correct waveform of $x(t)$ is

(A) 

(B) 

(C) 

(D) 

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EXCERCISE 1.3

MCQ 1.3.1 The period of signal $x(t) = 14 + 50\cos 60t$ is

(A) $\frac{\pi}{30}$ sec

(B) 60π sec

(C) $\frac{1}{60\pi}$ sec

(D) Not periodic

MCQ 1.3.2 The period of signal $x(t) = 10\sin 5t - 4\cos 7t$ is

(A) $\frac{24\pi}{35}$

(B) $\frac{4\pi}{35}$

(C) 2π

(D) Not periodic

MCQ 1.3.3 The period of signal $x(t) = 5t - 2\cos 5000\pi t$ is

(A) 0.96 ms

(B) 1.4 ms

(C) 0.4 ms

(D) Not periodic

MCQ 1.3.4 The period of signal $x(t) = 4\sin 3t + 3\sin\sqrt{t}$ is

(A) $\frac{2\pi}{3}$ sec

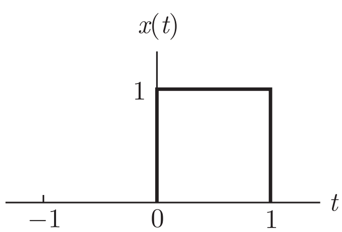
(B) $\frac{2\pi}{\sqrt{3}}$ sec

(C) 2π sec

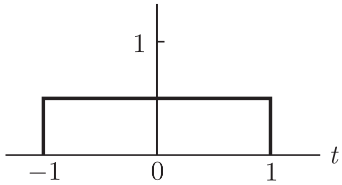
(D) Not periodic

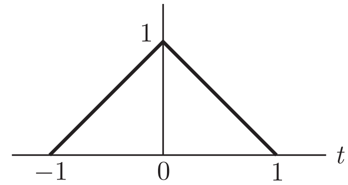
Statement for Q. 5 & 6

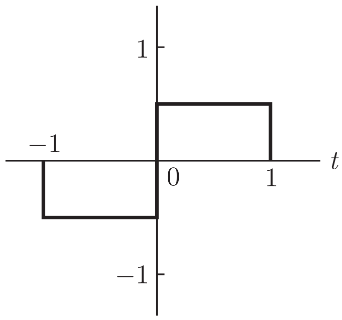
Consider the signal shown below

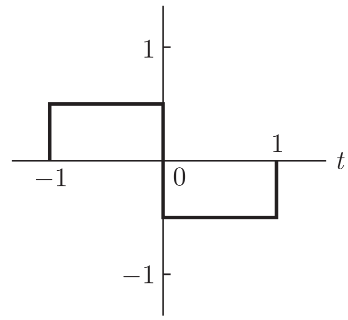


MCQ 1.3.5 The even part of signal is

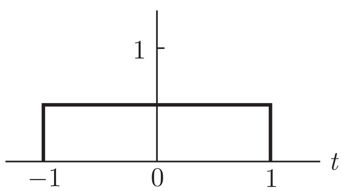
(A)


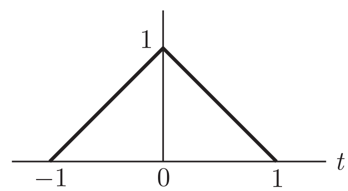
(B)


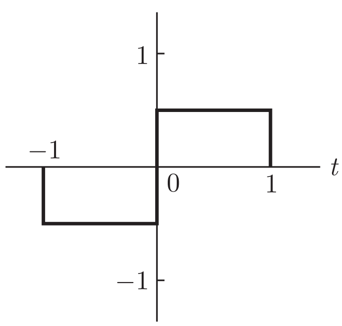
(C)


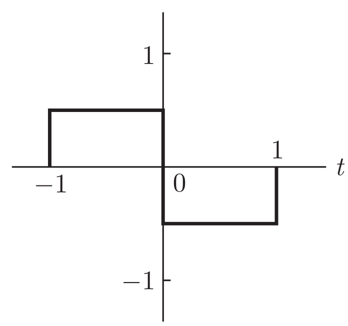
(D)


MCQ 1.3.6 The odd part of signal is

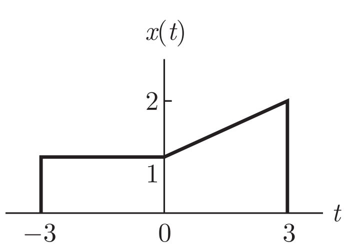
(A)


(B)


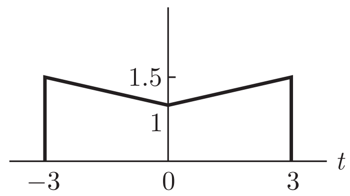
(C)


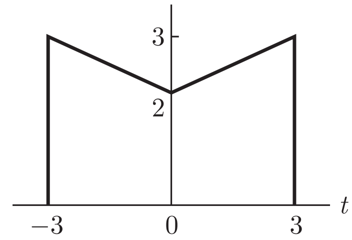
(D)


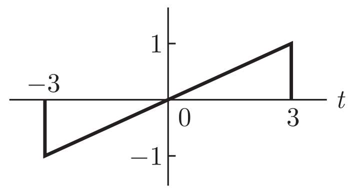
MCQ 1.3.7 Consider the function $x(t)$ shown in figure

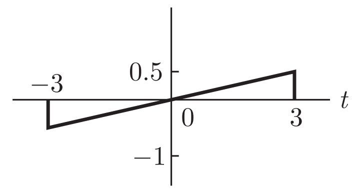


The even part of $x(t)$ is

(A) 

(B) 

(C) 

(D) 

MCQ 1.3.8 The signal $x(t) = e^{-4t}u(t)$ is a

(A) power signal with power $P_x = 1/4$ (B) power signal with power $P_x = 0$

(C) energy signal with energy $E_x = 1/4$ (D) energy signal with energy $E_x = 0$

MCQ 1.3.9 The signal $x(t) = e^{j(2t+\frac{\pi}{4})}$ is a

(A) power signal with $P_x = 1$ (B) power signal with $P_x = 2$

(C) energy signal with $E_x = 2$ (D) energy signal with $E_x = 1$

MCQ 1.3.10 The raised cosine pulse $x(t)$ is defined as

$$x(t) = \begin{cases} \frac{1}{2}(\cos\omega t + 1), & -\frac{\pi}{\omega} \le t \le \frac{\pi}{\omega} \\ 0, & \text{otherwise} \end{cases}$$

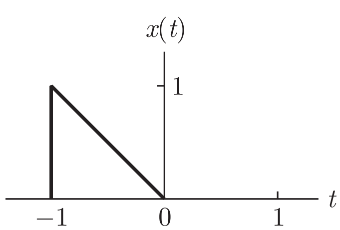
The total energy of $x(t)$ is

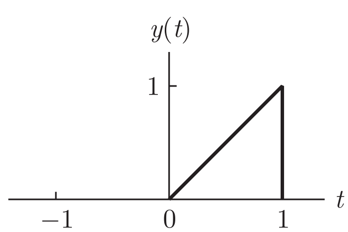
(A) $\frac{3\pi}{4\omega}$ (B) $\frac{3\pi}{8\omega}$

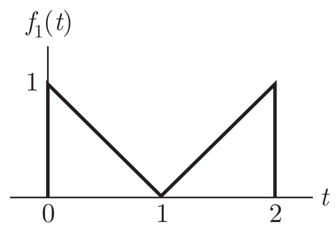
(C) $\frac{3\pi}{\omega}$ (D) $\frac{3\pi}{2\omega}$

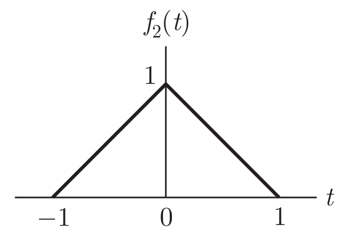
Statement for Q. 11 -14 :

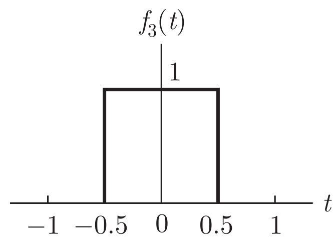
Consider the six signals shown in figure below.

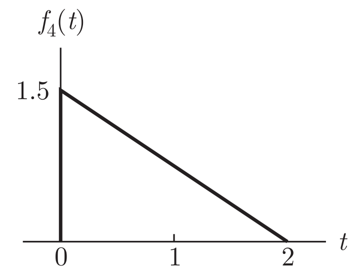












MCQ 1.3.11 The signal $f_1(t)$ can be expressed as

(A) $x(t-1)+y(t+1)$ (B) $x(t-1)+y(t-1)$

(C) $x(t+1)+y(t+1)$ (D) $x(t+1)+y(t-1)$

MCQ 1.3.12 The signal $f_2(t)$ can be expressed as

(A) $x(t-1)+y(t+1)$ (B) $x(t-1)+y(t-1)$

(C) $x(t+1)+y(t+1)$ (D) $x(t+1)+y(t-1)$

MCQ 1.3.13 The signal $f_3(t)$ can be expressed as

(A) $x(t-0.5)+y(t+0.5)$ (B) $x(t+0.5)+y(t+0.5)$

(C) $x(t-0.5)+y(t-0.5)$ (D) $x(t+0.5)+y(t-0.5)$

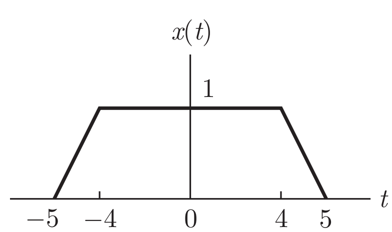
MCQ 1.3.14 The signal $f_4(t)$ can be expressed as

(A) $1.5x(2t-2)$ (B) $1.5x(\frac{t-1}{2})$

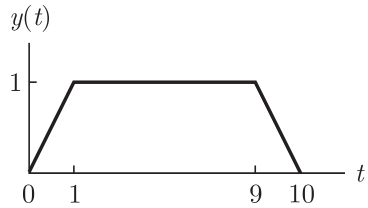
(C) $1.5x(2t-1)$ (D) $1.5x(\frac{t}{2}-1)$

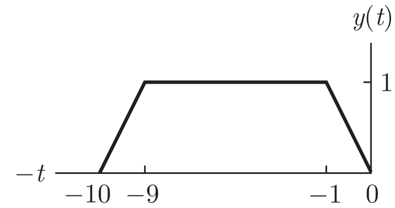
Statement for Q. 15-19 :

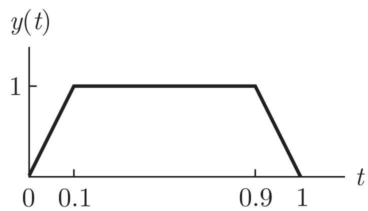
The signal $x(t)$ is depicted in figure below :

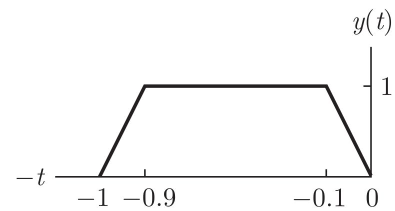


MCQ 1.3.15 The trapezoidal pulse $y(t)$ is related to the $x(t)$ as $y(t) = x(10t - 5)$. The sketch of $y(t)$ is

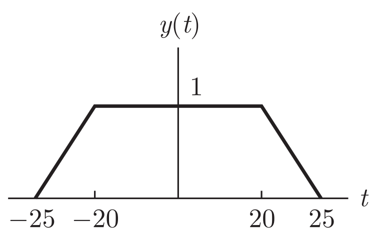
(A)


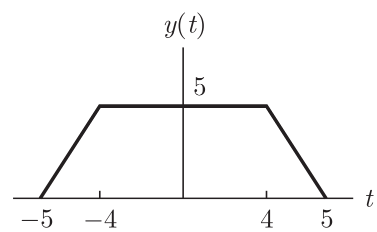
(B)


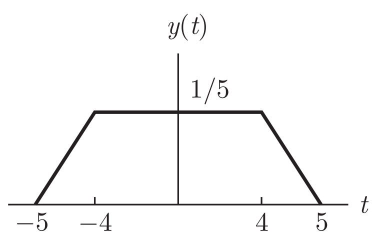
(C)


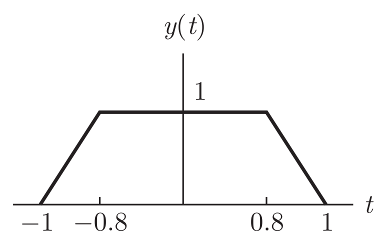
(D)


MCQ 1.3.16 The trapezoidal pulse $x(t)$ is time scaled producing $y(t) = x(5t)$. The sketch for $y(t)$ is

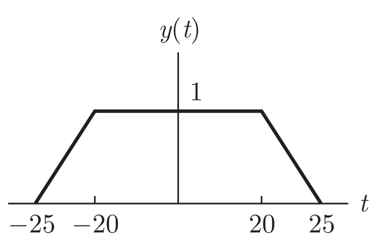
(A)


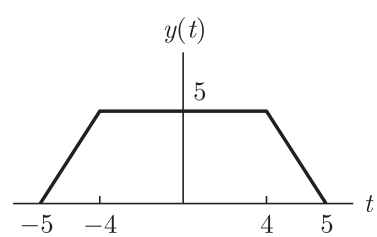
(B)


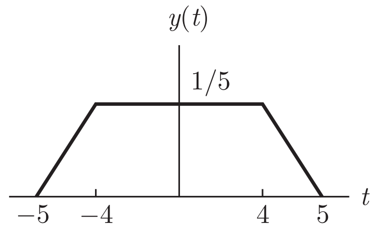
(C)


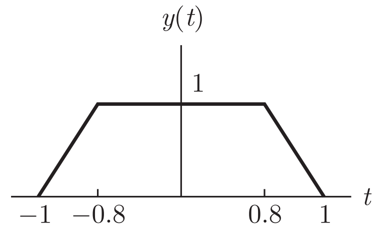
(D)


MCQ 1.3.17 The trapezoidal pulse $x(t)$ is time scaled producing $y(t) = x\left(\frac{t}{5}\right)$. The sketch for $y(t)$ is

(A)


(B)


(C) 

(D) 

MCQ 1.3.18 The trapezoidal pulse $x(t)$ is applied to a differentiator, defined by $y(t) = \frac{dx(t)}{dt}$. The total energy of $y(t)$ is

(A) 0 (B) 1

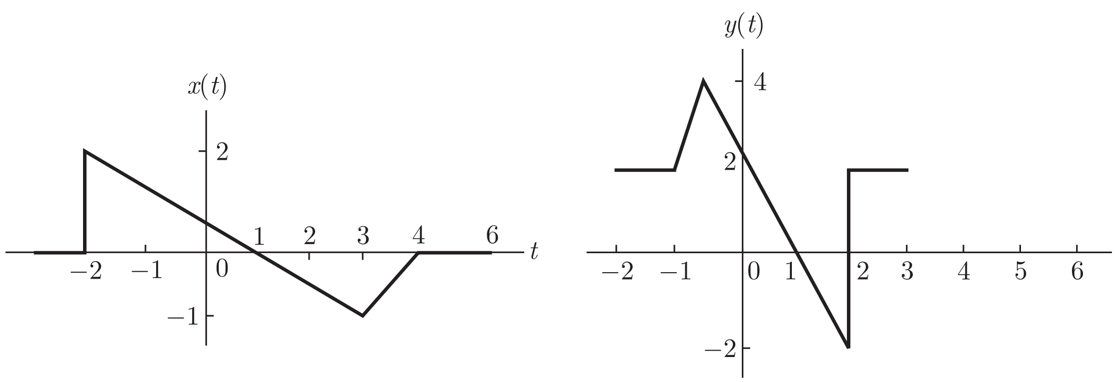
(C) 2 (D) 3

MCQ 1.3.19 The total energy of $x(t)$ is

(A) 0 (B) 13

(C) 13/3 (D) 26/3

MCQ 1.3.20 Consider the two signal shown in figure below.



The signal $y(t)$ can be represented as

(A) $2x\left(\frac{1}{2}t+2\right)+2$ (B) $2x(2t-2)-2$

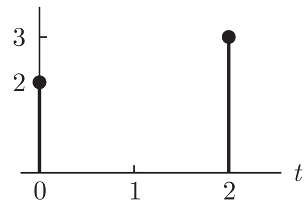
(C) $-2x(-2t+2)+2$ (D) $-2x\left(-\frac{1}{2}t+4\right)+2$

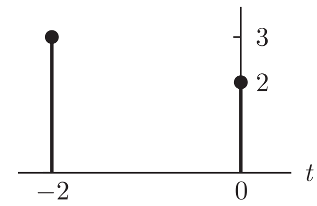
MCQ 1.3.21 The numerical value of integral $\int_{-1}^{8}[\delta(t+3)-2\delta(4t)]\,dt$ is

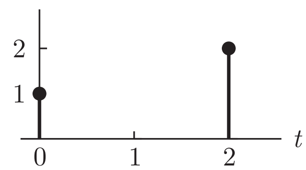
(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

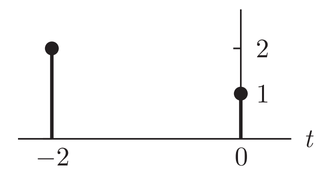
(C) 2 (D) -2

MCQ 1.3.22 The graph of function $x(t) = 2\delta(2t) + 6\delta(3(t-2))$ is

(A) 

(B) 

(C) 

(D) 

MCQ 1.3.23 The function $\int_{-\infty}^{\infty} x(\tau)[\delta(\tau-2) + \delta(\tau+2)]\,d\tau$ is equal to

(A) $x(2) + x(-2)$

(B) $\dfrac{x(2) + x(-2)}{2}$

(C) $2x(2) + 2x(-2)$

(D) None of these

MCQ 1.3.24 The value of the function $\int_{-\infty}^{\infty} \delta(at-b)\sin^2(t-4)\,dt$ where $a > 0$, is

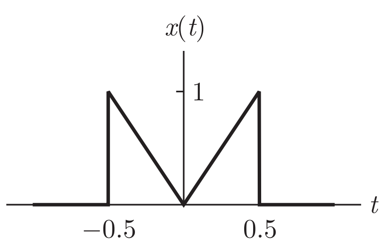
(A) 1

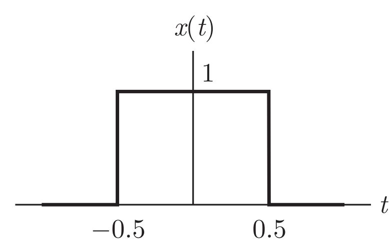
(B) $\dfrac{\sin^2(\frac{a}{b}-4)}{b}$

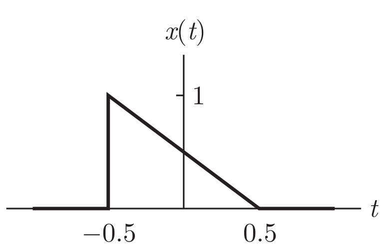
(C) 0

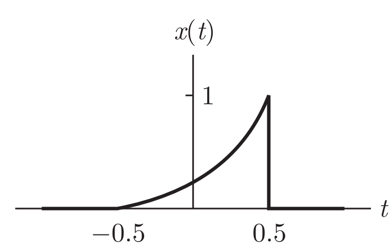
(D) $\dfrac{\sin^2(\frac{b}{a}-4)}{a}$

MCQ 1.3.25 Consider the function $x(t) = u\left(t+\frac{1}{2}\right)\text{ramp}\left(\frac{1}{2}-t\right)$. The graph of $x(t)$ is

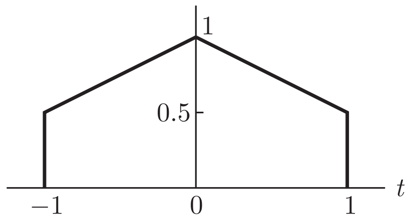
(A) 

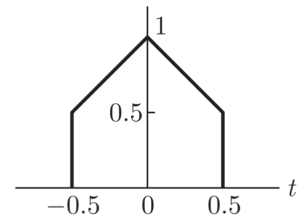
(B) 

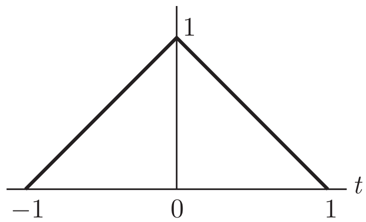
(C) 

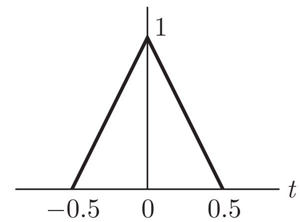
(D) 

MCQ 1.3.26 Consider the signal $x(t) = \text{rect}(t)\,\text{tri}(t)$. The graph of $x(t)$ is

(A)


(B)


(C)


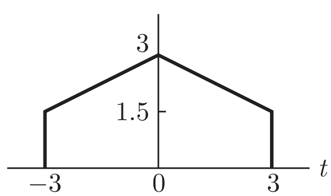
(D)


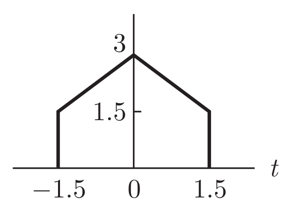
MCQ 1.3.27 A signal is defined as $x(t) = 4\text{tri}(t)$. The value of $x(\frac{1}{2})$ is

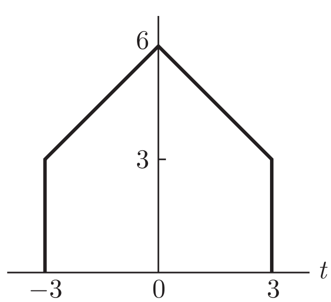
(A) 1/2 (B) 1

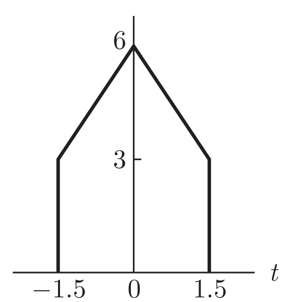
(C) 2 (D) 0

MCQ 1.3.28 Consider the signal $x(t) = 3\text{tri}\left(\frac{2t}{3}\right) + 3\text{rect}\left(\frac{t}{3}\right)$. The graph of $x(t)$ is

(A)


(B)


(C)


(D)


Statement for Q. 29 - 30 :

Let the CT unit impulse function be defined by

$$\delta(x) = \lim_{\alpha \to 0}\left(\frac{1}{\alpha}\right)\text{tri}\left(\frac{x}{\alpha}\right),\ a > 0$$

The function $\delta(x)$ has an area of one regardless the value of α

MCQ 1.3.29 What is the area of the function $\delta(4x)$?

(A) 1 (B) $\frac{1}{4}$

(C) 4 (D) 2

MCQ 1.3.30 What is the area of the function $\delta(-6x)$?

(A) 1 (B) 1/6

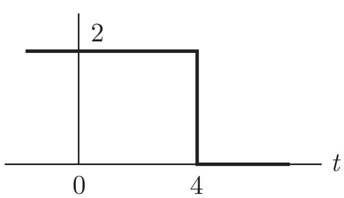
(C) 4 (D) 2

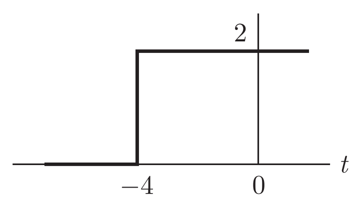
MCQ 1.3.31 A signal $x(t)$ is defined as $x(t) = 2\text{tri}[2(t-1)] + 6\text{rect}(\frac{t}{4})$. The value of $x(\frac{3}{2})$ is

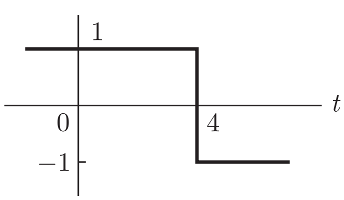
(A) 4 (B) 5

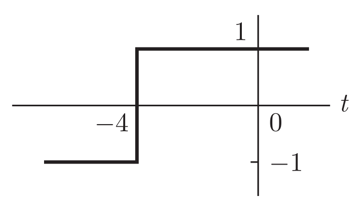
(C) 6 (D) 7

MCQ 1.3.32 A function is defined as $x(t) = 1 + \text{sgn}(4-t)$. The graph of $x(t)$ is

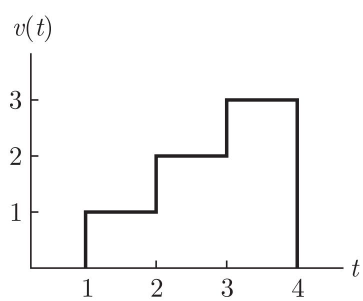
(A)


(B)


(C)


(D)


MCQ 1.3.33 Consider the voltage waveform shown below The equation for $v(t)$ is



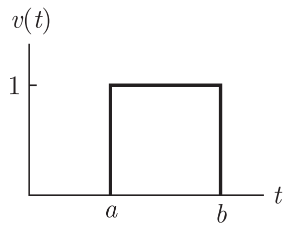
(A) $u(t-1) + u(t-2) + u(t-3)$

(B) $u(t-1) + 2u(t-2) + 3u(t-3)$

(C) $u(t-1) + u(t-2) + u(t-2)$

(D) $u(t-1) + u(t-2) + u(t-3) - 3u(t-4)$

MCQ 1.3.34 Consider the following function for the rectangular voltage pulse shown below



(1) $v(t) = u(a-t) \times u(t-b)$

(2) $v(t) = u(b-t) \times u(t-a)$

(3) $v(t) = u(t-a) - u(t-b)$

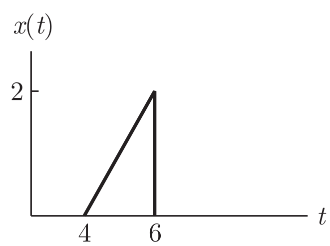
The function that describe the pulse are

(A) 1 and 2 (B) 2 and 3

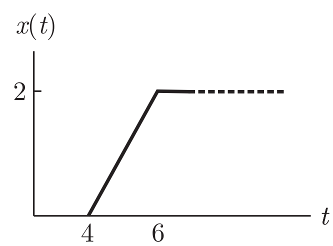
(C) 1 and 3 (D) all

MCQ 1.3.35 A signal is described by $x(t) = r(t-4) - r(t-6)$, where $r(t)$ is a ramp function starting at $t = 0$. The signal $x(t)$ is represented as

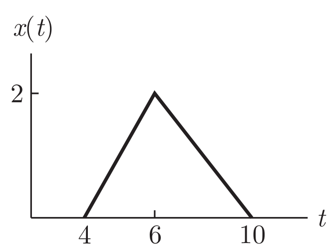
(A)



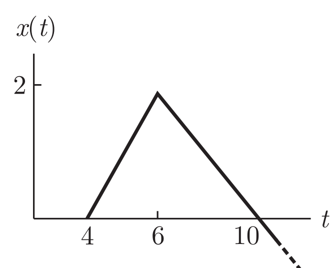
(B)



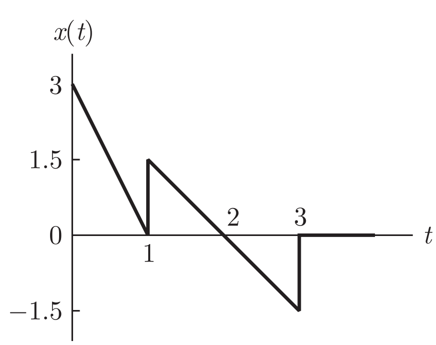
(C)



(D)



MCQ 1.3.36 For the waveform shown in figure the equation is



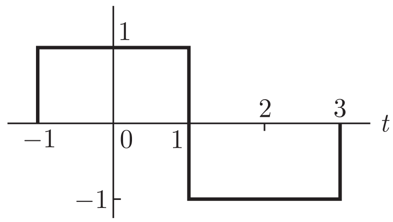
(A) $-3tu(t) + 1.5(t-2)u(t-1) + 1.5(t-3)u(t-3)$

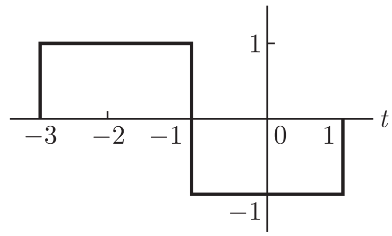
(B) $3(2-t)u(t) + 1.5(t-2)u(t-1) + 1.5(t-3)u(t-3)$

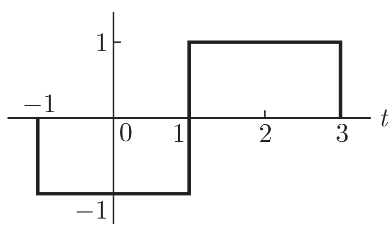
(C) $3(1-t)u(t)+1.5tu(t-1)+1.5(t-2)u(t-3)$

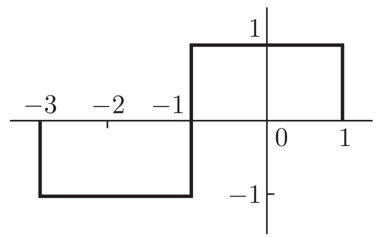
(D) None of these

MCQ 1.3.37 For the signal $x(t)=u(t+1)-2u(t-1)+u(t-3)$, the correct wave form is

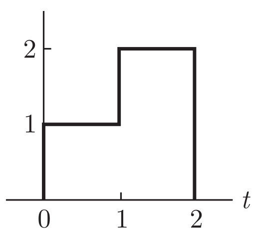
(A) 

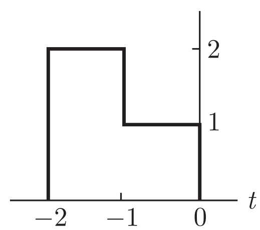
(B) 

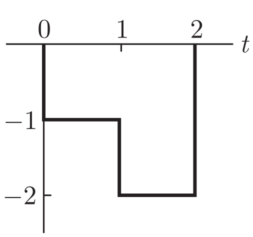
(C) 

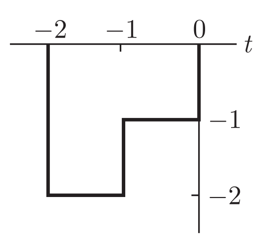
(D) 

MCQ 1.3.38 For the signal $x(t)=u(t)+u(t+1)-2u(t+2)$, the correct waveform is

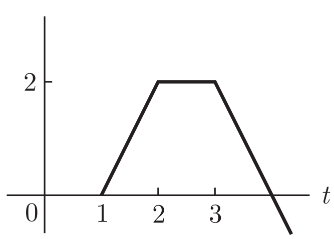
(A) 

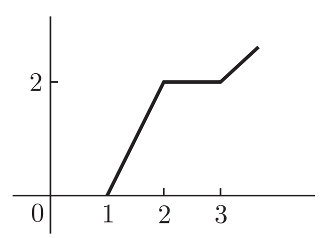
(B) 

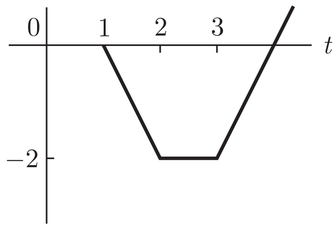
(C) 

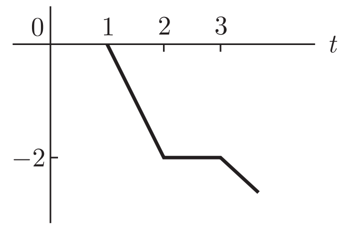
(D) 

MCQ 1.3.39 For the signal $x(t)=2(t-1)u(t-1)-2(t-2)u(t-2)+2(t-3)u(t-3)$ the correct waveform is

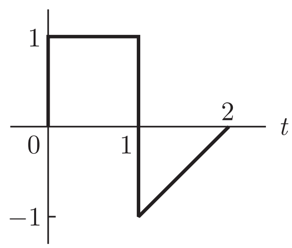
(A) 

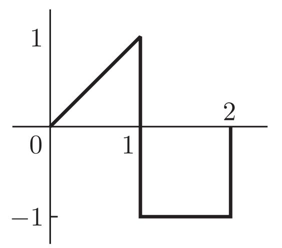
(B) 

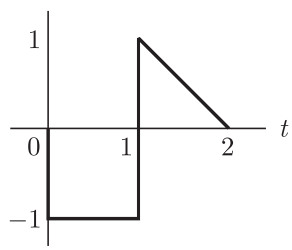
(C) 

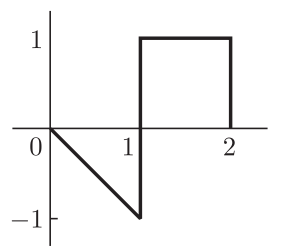
(D) 

MCQ 1.3.40 For the signal $x(t) = (t+1)\,u(t-1) - tu(t) - u(t-2)$ the correct waveform is

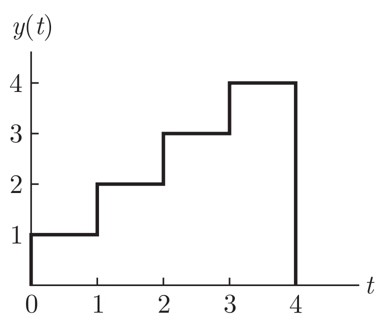
(A) 

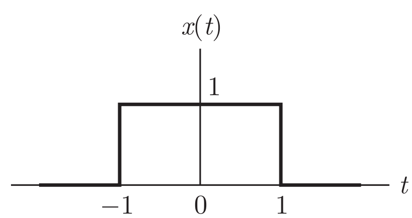
(B) 

(C) 

(D) 

MCQ 1.3.41 Consider the two signal shown in figure





The signal $y(t)$ can be explained as

(A) $x(\frac{1}{2}t-1) + x(\frac{2}{3}t-\frac{5}{3}) + x(t-3) + x(2t-7)$

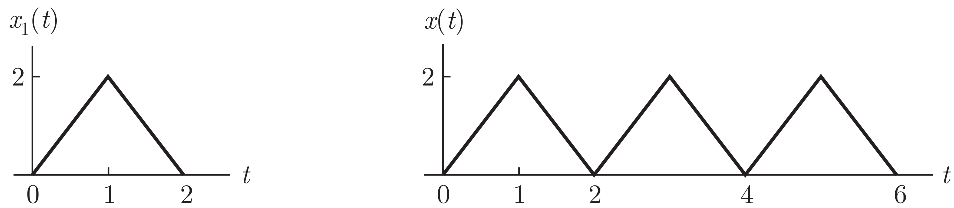
(B) $x(2t+1) + x(\frac{3}{2}t+\frac{5}{3}) + x(t+3) + x(2t+7)$

(C) $x(\frac{1}{2}t+1) + x(\frac{2}{3}t+\frac{5}{3}) + (t+3) + x(2t+7)$

(D) $x(2t-1) + x(\frac{3}{2}t-\frac{5}{3}) + x(t-3) + x(2t-7)$

Statement for Q. 42-43 :

Consider the triangular pulses and the triangular wave of figure



MCQ 1.3.42 The mathematical function for $x_1(t)$ is

(A) $2tu(t) - 4(t+1)u(t-1) + 2(t+2)u(t-2)$

(B) $2tu(t) - 4(t-1)u(t-1) + 2(t-2)u(t-2)$

(C) $2tu(t) - 4(t-1)u(t+1) + 2(t-2)u(t+2)$

(D) None of the above

MCQ 1.3.43 The mathematical function for waveform $x(t)$ is

(A) $\sum_{k=0}^{\infty} x_1(t+2k)$

(B) $\sum_{k=-\infty}^{\infty} x_1(t-2k)$

(C) $\sum_{k=0}^{\infty} x_1(t-2k)$

(D) $\sum_{k=-\infty}^{\infty} x_1(t+2k)$

Here, $T_0 = 2$, therefore

$$x(t) = \sum_{k=-\infty}^{\infty} x_1(t-2k)$$

****

EXCERCISE 1.4

MCQ 1.4.1 IES EC 2009

A function of one or more variable which conveys information on the nature of physical phenomenon is called

(A) Noise (B) Interference

(C) System (D) Signal

MCQ 1.4.2 GATE IN 2006

The Fourier series for a periodic signal is given as

$$x(t) = \cos(1.2\pi t) + \cos(2\pi t) + \cos(2.8\pi t)$$

The fundamental frequency of the signal is

(A) 0.2 Hz (B) 0.6 Hz

(C) 1.0 Hz (D) 1.4 Hz

MCQ 1.4.3 GATE IN 2007

Consider the periodic signal $x(t) = (1 + 0.5\cos 40\pi t)\cos 200\pi t$, where t is in seconds. Its fundamental frequency, in Hz, is

(A) 20 (B) 40

(C) 100 (D) 200

MCQ 1.4.4 GATE IN 2009

The fundamental period of $x(t) = 2\sin 2\pi t + 3\sin 3\pi t$, with t expressed in seconds, is

(A) 1 s (B) 0.67 s

(C) 2 s (D) 3 s

MCQ 1.4.5 IES EC 1999

The period of the function $\cos[\pi/4(t-1)]$ is

(A) 1/8 s (B) 8 s

(C) 4 s (D) 1/4 s

MCQ 1.4.6 IES EC 2001

If $x_1(t) = 2\sin\pi t + \cos 4\pi t$ and $x_2(t) = \sin 5\pi t + 3\sin 13\pi t$, then

(A) x_1 and x_2 both are periodic

(B) x_1 and x_2 both are not periodic

(C) x_1 is periodic, but x_2 is not periodic

(D) x_1 is not periodic, but x_2 is periodic

MCQ 1.4.7 IES EC 2003

The sum of two or more arbitrary sinusoids is

(A) Always periodic

(B) Periodic under certain conditions

(C) Never periodic

(D) Periodic only if all the sinusoids are identical in frequency and phase

MCQ 1.4.8
IES EC 2004

Which one of the following must be satisfied if a signal is to be periodic for $-\infty < t < \infty$?

(A) $x(t+T_0) = x(t)$

(B) $x(t+T_0) = dx(t)/dt$

(C) $x(t+T_0) = \int_t^{T_0} x(t)\,dt$

(D) $x(t+T_0) = x(t) + kT_0$

MCQ 1.4.9
IES EC 2007

Consider two signals $x_1(t) = e^{j20t}$ and $x_2(t) = e^{(-2+j)t}$. Which one of the following statements is correct ?

(A) Both $x_1(t)$ and $x_2(t)$ are periodic

(B) $x_1(t)$ is periodic but $x_2(t)$ is not periodic

(C) $x_2(t)$ is periodic but $x_1(t)$ is not periodic

(D) Neither $x_1(t)$ nor $x_2(t)$ is periodic

MCQ 1.4.10
IES EC 2008

Which one of the following function is a periodic one ?

(A) $\sin(10\pi t) + \sin(20\pi t)$

(B) $\sin(10t) + \sin(20\pi t)$

(C) $\sin(10\pi t) + \sin(20t)$

(D) $\sin(10t) + \sin(25\pi t)$

MCQ 1.4.11
GATE EE 2010

The period of the signal $x(t) = 8\sin\left(0.8\pi t + \frac{\pi}{4}\right)$ is

(A) 0.4π s

(B) 0.8π s

(C) 1.25 s

(D) 2.5 s

MCQ 1.4.12
IES EC 2009

A signal $x_1(t)$ and $x_2(t)$ constitute the real and imaginary parts respectively of a complex valued signal $x(t)$. What form of waveform does $x(t)$ possess ?

(A) Real symmetric

(B) Complex symmetric

(C) Asymmetric

(D) Conjugate symmetric

MCQ 1.4.13
IES EC 1991

If from the function $f(t)$ one forms the function, $\Psi(t) = f(t) + f(-t)$, then $\Psi(t)$ is

(A) even

(B) odd

(C) neither even nor odd

(D) both even and odd

MCQ 1.4.14
IES EC 2001

The signal $x(t) = A\cos(\omega t + \phi)$ is

(A) an energy signal

(B) a power signal

(C) an energy as well as a power signal

(D) neither an energy nor a power signal

MCQ 1.4.15
IES EC 2007

Which one of the following is the mathematical representation for the average power of the signal $x(t)$?

(A) $\frac{1}{T}\int_0^T x(t)\,dt$

(B) $\frac{1}{T}\int_0^T x^2(t)\,dt$

(C) $\frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$

(D) $\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$

MCQ 1.4.16
IES EC 2007

Which one of the following is correct ?

Energy of a power signal is

(A) finite (B) zero

(C) infinite (D) between 1 and 2

MCQ 1.4.17
GATE EC 2005

The power in the signal $s(t) = 8\cos(20\pi - \frac{\pi}{2}) + 4\sin(15\pi t)$ is

(A) 40 (B) 41

(C) 42 (D) 82

MCQ 1.4.18
GATE EE 2006

Which of the following is true ?

(A) A finite signal is always bounded

(B) A bounded signal always possesses finite energy

(C) A bounded signal is always zero outside the interval $[-t_0, t_0]$ for some t_0

(D) A bounded signal is always finite

MCQ 1.4.19
GATE EC 2001

If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to

(A) 1 (B) $E/2$

(C) $2E$ (D) $4E$

MCQ 1.4.20
IES EC 2001

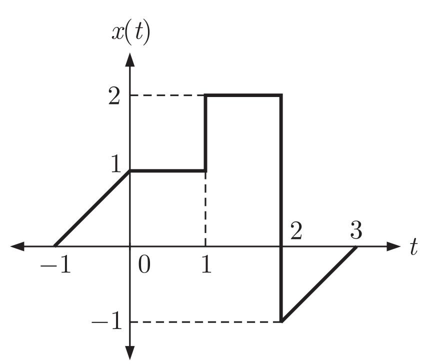
If a function $f(t)\,u(t)$ is shifted to right side by t_0, then the function can be expressed as

(A) $f(t-t_0)\,u(t)$ (B) $f(t)\,u(t-t_0)$

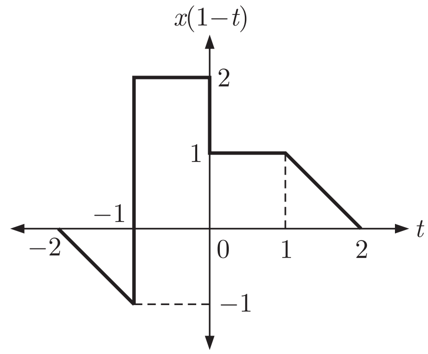
(C) $f(t-t_0)\,u(t-t_0)$ (D) $f(t+t_0)\,u(t+t_0)$

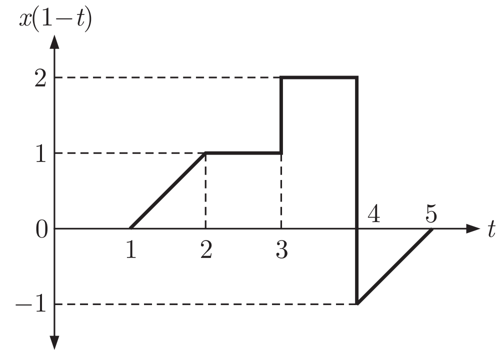
MCQ 1.4.21
IES EC 1999

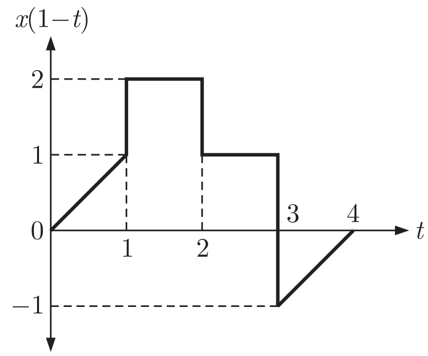
If a plot of signal $x(t)$ is as shown in the figure

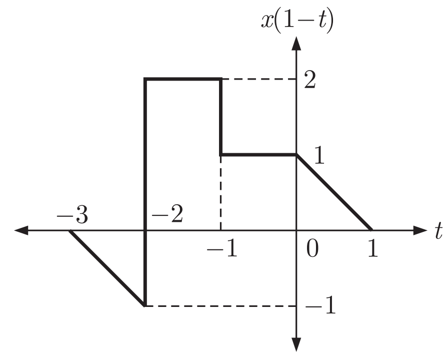


then the plot of the signal $x(1-t)$ will be

(A) 

(B) 

(C) 

(D) 

MCQ 1.4.22 A signal $v[n]$ is defined by

IES EC 2005

$$v[n] = \begin{cases} 1 & ;\ n = 1 \\ -1 & ;\ n = -1 \\ 0 & ;\ n = 0 \text{ and } |n| > 1 \end{cases}$$

Which is the value of the composite signal defined as $v[n] + v[-n]$?

(A) 0 for all integer values of n

(B) 2 for all integer values of n

(C) 1 for all integer values of n

(D) -1 for all integer values of n

MCQ 1.4.23 Which one of the following relations is not correct ?

IES EC 2011

(A) $f(t)\delta(t) = f(0)\delta(t)$

(B) $\int_{-\infty}^{\infty} f(t)\delta(\tau)\,d\tau = 1$

(C) $\int_{-\infty}^{\infty}\delta(\tau)\,d\tau = 1$

(D) $f(t)\delta(t-\tau) = f(\tau)\delta(t-\tau)$

MCQ 1.4.24 The Dirac delta function $\delta(t)$ is defined as

GATE EC 2006

(A) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(B) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(C) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

(D) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

MCQ 1.4.25
GATE EC 2001

Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt$ is

(A) 1
(B) -1
(C) 0
(D) $\frac{\pi}{2}$

MCQ 1.4.26
GATE IN 2010

The Integral $\int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{6}\right) 6\sin(t)\,dt$ evaluates to

(A) 6
(B) 3
(C) 1.5
(D) 0

MCQ 1.4.27
GATE IN 2011

The integral $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} t^2 e^{-t^2/2}\delta(1-2t)\,dt$ is equal to

(A) $\frac{1}{8\sqrt{2\pi}}e^{-1/8}$
(B) $\frac{1}{4\sqrt{2\pi}}e^{-1/8}$
(C) $\frac{1}{\sqrt{2\pi}}e^{-1/2}$
(D) 1

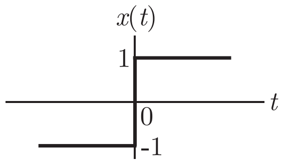
MCQ 1.4.28
IES EC 1995

Double integration of a unit step function would lead to

(A) an impulse
(B) a parabola
(C) a ramp
(D) a doublet

MCQ 1.4.29
GATE EC 2005

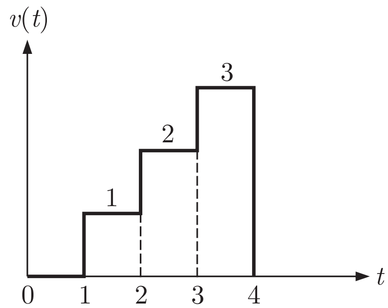
The function $x(t)$ is shown in the figure. Even and odd parts of a unit step function $u(t)$ are respectively,



(A) $\frac{1}{2}, \frac{1}{2}x(t)$
(B) $-\frac{1}{2}, \frac{1}{2}x(t)$
(C) $\frac{1}{2}, -\frac{1}{2}x(t)$
(D) $-\frac{1}{2}, -\frac{1}{2}x(t)$

MCQ 1.4.30 IES EC 1991

The expression for the wave form in terms of step function is given by



(A) $v(t) = u(t-1) - u(t-2) + u(t-3)$

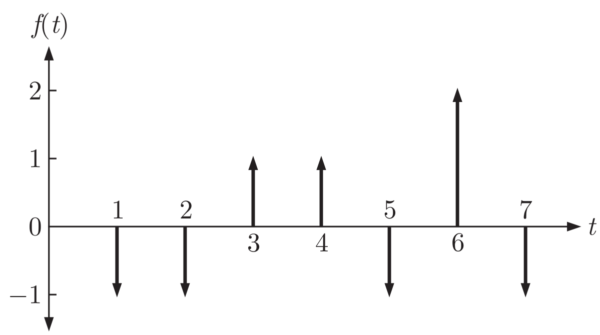
(B) $v(t) = u(t-1) + u(t-2) + u(t-3)$

(C) $v(t) = u(t-1) + u(t-2) - u(t-3)$

(D) $v(t) = u(t-1) + u(t-2) + u(t-3) - 3u(t-4)$

MCQ 1.4.31 IES EC 1991

The impulse train shown in the figure represents the second derivative of a function $f(t)$. The value of $f(t)$ is



(A) $-tu(t-1) - tu(t-2) + tu(t-3) + tu(t-4) - tu(t-5) + 2tu(t-6) - tu(t-7)$

(B) $-tu(t-1) - tu(t-2) - tu(t-3) - tu(t-4) + tu(t-5)$

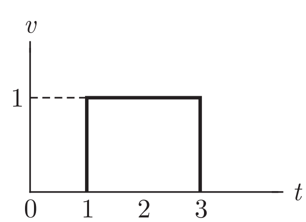
(C) $tu(t-3) + tu(t-4) + 2tu(t-6)$

(D) $tu(t+1) + tu(t+2) + tu(t+3) + tu(t+4) + tu(t+5) + 2tu(t+6) + tu(t+7)$

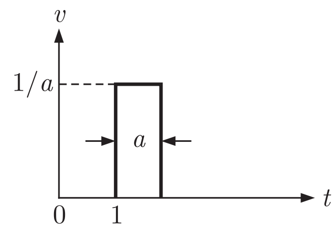
MCQ 1.4.32 IES E C 1997

Match List I with List II and select the correct answer using the codes given below the Lists:

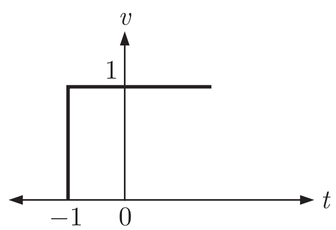
List I **List II**

A. 

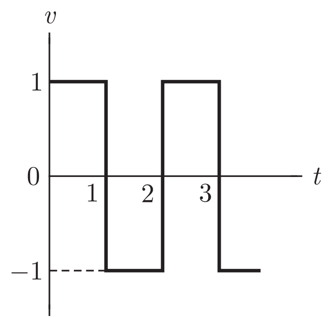
1. $v(t) = u(t+1)$

B. 

2. $v(t) = u(t-1) - 2u(t-1) + 2u(t-2) - 2u(t-3) + ...$

C. 

3. $v(t) = u(t-1) - u(t-3)$

D. 

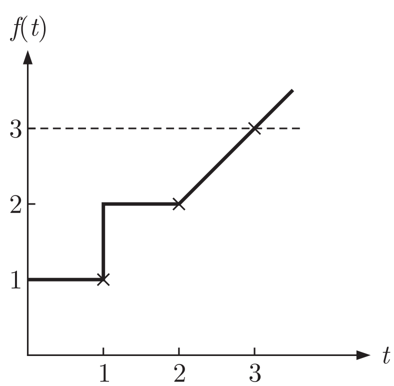
4. $\underset{a \to 0}{\text{Lim}}\, v(t) = \delta(t-1)$

Codes :

	A	B	C	D
(A)	1	2	3	4
(B)	3	4	1	2
(C)	4	3	2	1
(D)	4	3	1	2

MCQ 1.4.33 Consider the following waveform diagram

IES EC 2004

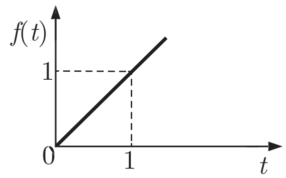
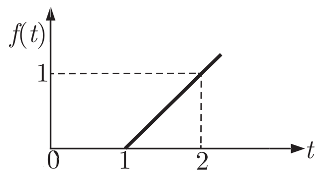
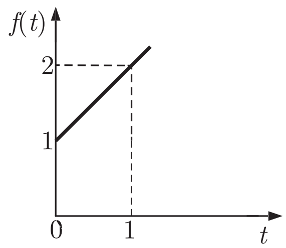
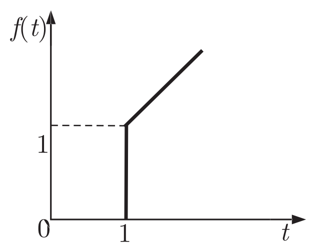


Which one of the following gives the correct description of the waveform shown in the above diagram ?

(A) $u(t) + u(t-1)$
(B) $u(t) + (t-1)\,u(t-1)$
(C) $u(t) + u(t-1) + (t-2)\,u(t-2)$
(D) $u(t) + (t-2)\,u(t-2)$

MCQ 1.4.34
GATE EE 1994

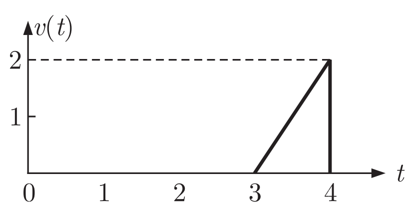
Match the waveforms on the left-hand side with the correct mathematical description listed on the right hand side.

Waveform	$f(t)$
(P) 	(1) $tu(t-1)$
(Q) 	(2) $(t+1)\,u(t-1)$
	(3) $tu(t)$
(R) 	(4) $(t+1)\,u(t)$
	(5) $(t-1)\,u(t)$
(S) 	(6) $(t-1)\,u(t-1)$

(A) P-1, Q-3, R-4, S-2
(B) P-3, Q-6, R-4, S-1
(C) P-1, Q-6, R-2, S-4
(d) P-2, Q-3, R-4, S-1

MCQ 1.4.35
IES EC 2005

In the graph shown below, which one of the following express $v(t)$?



(A) $(2t+6)\,[u(t-3)+2u(t-4)]$
(B) $(-2t-6)\,[u(t-3)+u(t-4)]$
(C) $(-2t+6)\,[u(t-3)+u(t-4)]$
(D) $(2t-6)\,[u(t-3)-u(t-4)]$

****

SOLUTIONS 1.1

SOLUTIONS 1.2

SOL 1.2.1 Option (A) is correct.

Period of $\sin(4\pi t)$, $T_1 = \frac{2\pi}{4\pi} = \frac{1}{2}$

Period of $\cos(3\pi t)$, $T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}$

Ratio, $\frac{T_1}{T_2} = \frac{1/2}{2/3} = \frac{3}{4}$ (rational)

So, the signal $x(t)$ is periodic.

Period of $x(t)$, $T = \text{LCM}(T_1, T_2) = \text{LCM}\left(\frac{1}{2}, \frac{2}{3}\right) = 2\,\text{sec}$

Alternate Method :

$$\frac{T_1}{T_2} = \frac{m}{n}$$

Fundamental period of $x(t)$

$$T = nT_1 = mT_2$$

Here $\frac{T_1}{T_2} = \frac{3}{4} = \frac{m}{n}$

Thus $m = 3, n = 4$

Period of $x(t)$, $T = nT_1 = 4 \times \frac{1}{2} = 2\,\text{sec}$

or $T = mT_2 = 3 \times \frac{2}{3} = 2\,\text{sec}$

SOL 1.2.2 Option (D) is correct.

Period of $\sin(4\pi t)$, $T_1 = \frac{2\pi}{4\pi} = \frac{1}{2}$

Period of $\cos(10t)$, $T_2 = \frac{2\pi}{10} = \frac{\pi}{5}$

Here $\frac{T_1}{T_2} = \frac{1/2}{\pi/5} = \frac{5}{2\pi}$ (not rational)

Since the ratio T_1/T_2 is not rational, $x(t)$ is not periodic.

SOL 1.2.3 Option (A) is correct.

For $x_1(t)$:

Period of $\sin(8\pi t)$, $T_1 = \frac{2\pi}{8\pi} = \frac{1}{4}$

Period of $\cos(6\pi t)$, $T_2 = \frac{2\pi}{6\pi} = \frac{1}{3}$

Now $\frac{T_1}{T_2} = \frac{1/4}{1/3} = \frac{3}{4}$ (rational)

Ratio T_1/T_2 is a rational number, therefore $x_1(t)$ is a periodic signal.

For $x_2(t)$:

Period of $\sin(8\pi t)$, $T_1 = \frac{2\pi}{8\pi} = \frac{1}{4}$

Period of $\cos(20t)$, $T_2 = \frac{2\pi}{20} = \frac{\pi}{10}$

Check for periodicity $\frac{T_1}{T_2} = \frac{1/4}{\pi/10} = \frac{5}{2\pi}$ (not rational)

Ratio T_1/T_2 is not rational, therefore $x_2(t)$ is not periodic.

SOL 1.2.4 Option (C) is correct.

$$x(t) = \sin\left[\left(\frac{2\pi}{5}\right)t\right]\cos\left[\left(\frac{4\pi}{3}\right)t\right]$$

$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$

$$= \frac{1}{2}\left[\sin\left(\frac{2\pi}{5} - \frac{4\pi}{3}\right)t + \sin\left(\frac{2\pi}{5} + \frac{4\pi}{3}\right)t\right]$$

$$= \frac{1}{2}\left[\sin\left(\frac{-14\pi}{15}\right)t + \sin\left(\frac{26\pi}{15}\right)t\right]$$

$$= x_1(t) + x_2(t)$$

Period of $x_1(t)$, $T_1 = \frac{2\pi}{(14\pi/15)} = \frac{15}{7}$

Period of $x_2(t)$, $T_2 = \frac{2\pi}{(26\pi/15)} = \frac{15}{13}$

$$\frac{T_1}{T_2} = \frac{15/7}{15/13} = \frac{13}{7} = \frac{m}{n} \text{ (rational)}$$

Here $m = 13$ and $n = 7$. Let period of $x(t)$ is T, then

$$T = mT_2 = nT_1$$

Thus, $T = 13 \times \frac{15}{13} = 15\,\text{sec}$

or $T = 7 \times \frac{15}{7} = 15\,\text{sec}$

Alternate Method :

Period of $x(t)$, $T = \text{LCM}(T_1, T_2)$

$$T = \text{LCM}\left(\frac{15}{7}, \frac{15}{13}\right)$$

$$= 15\,\text{sec}$$

SOL 1.2.5 Option (D) is correct

Period of $f_1(t)$, $T_1 = \frac{2\pi}{2\pi/3} = 3$ unit

$f_2(t)$ can be written as

$$f_2(t) = \frac{1}{2}\left[\sin\left(\frac{2\pi}{5} - \frac{4\pi}{3}\right)t + \sin\left(\frac{2\pi}{5} + \frac{4\pi}{3}\right)t\right]$$

$$= \frac{1}{2}\left[\sin\left(\frac{-14\pi}{15}\right)t + \sin\left(\frac{26\pi}{15}\right)t\right]$$

Let $f_2(t) = f_{21}(t) + f_{22}(t)$

Period of $f_{21}(t)$, $T_{21} = \frac{2\pi}{(14\pi/15)} = \frac{15}{7}$

Period of $f_{22}(t)$, $T_{22} = \frac{2\pi}{(26\pi/15)} = \frac{15}{13}$

Ratio, $\frac{T_{21}}{T_{22}} = \frac{15/7}{15/13} = \frac{13}{7}$ (rational)

So, $f_2(t)$ is periodic.

Period of $f_2(t)$, $T_2 = \text{LCM}(T_{21}, T_{22}) = \text{LCM}\left(\frac{15}{7}, \frac{15}{13}\right) = 15\,\text{sec}$

Period of $f_3(t)$, $T_3 = \frac{2\pi}{3}$ unit

$$f_4(t) = f_1(t) - 2f_3(t)$$

$$\text{Ratio } \frac{T_1}{T_3} = \frac{3}{2\pi/3} = \frac{9}{2\pi} \text{ (not rational)}$$

Therefore $f_4(t)$ is aperiodic.

Codes, P → 2, Q → 1, R → 4, S → 3

SOL 1.2.6 Option (C) is correct

Signal $\sin(10\pi t)\,u(t)$ is not periodic as it is defined for $t > 0$ only.

SOL 1.2.7 Option (B) is correct.

Let, $$g(t) = \underbrace{2\cos(10t+1)}_{g_1(t)} + \underbrace{\sin(4t-1)}_{g_2(t)}$$

Period of $g_1(t)$, $T_1 = \frac{2\pi}{10} = \frac{\pi}{5}$ sec

Period of $g_2(t)$, $T_2 = \frac{2\pi}{4} = \frac{\pi}{2}$ sec

Ratio, $\frac{T_1}{T_2} = \frac{\pi/5}{\pi/2} = \frac{2}{5}$ (rational)

Therefore, $g(t)$ is periodic

Period of $g(t)$, $T = \text{LCM}(T_1, T_2) = \text{LCM}\left(\frac{\pi}{5}, \frac{\pi}{2}\right) = \pi\,\text{sec}$

SOL 1.2.8 Option (D) is correct.

All the given signals are periodic.

Period of $x_1(t)$, $T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$

Period of $x_2(t)$, $T_2 = \frac{2\pi}{\pi} = 2$

Period of $x_3(t)$, $T_3 = \frac{2\pi}{4} = \frac{\pi}{2}$

None of the above signals is aperiodic.

SOL 1.2.9 Option (C) is correct.

Odd part of $g(t)$,

$$g_o(t) = \frac{1}{2}[g(t) - g(-t)]$$

$$g(-t) = \begin{cases} -t, & 0 \le -t < 1 \\ 0, & \text{elsewhere} \end{cases}$$

$$= \begin{cases} -t, & -1 < t \le 0 \\ 0, & \text{elsewhere} \end{cases}$$

So,

$$g_o(t) = \begin{cases} t/2, & -1 \le t < 0 \\ t/2, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$$

SOL 1.2.10 Option (B) is correct.

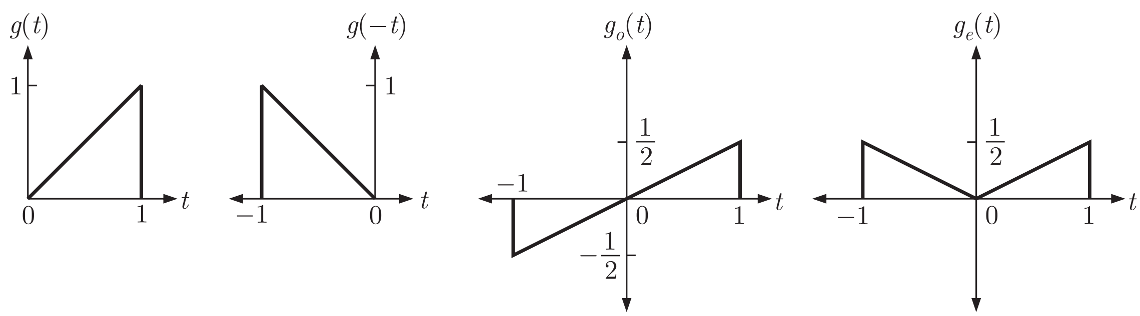
$$g(-t) = \begin{cases} -t, & -1 \le t < 0 \\ 0, & \text{elsewhere} \end{cases}$$

Even part

$$g_e(t) = \frac{1}{2}[g(t) + g(-t)]$$

$$= \begin{cases} -t/2, & -1 \le t < 0 \\ t/2, & 0 \le t < 1 \\ 0, & \text{elsewhere} \end{cases}$$

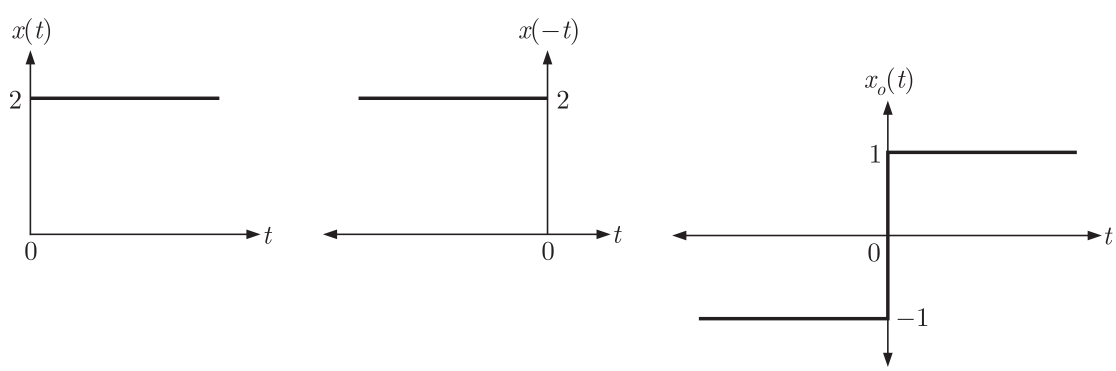
Graphically :



SOL 1.2.11 Option (B) is correct.

Odd part of $x(t)$, $\quad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

This is shown graphically as below :



The function $x_o(t)$ is unit signum function.

SOL 1.2.12 Option (B) is correct.

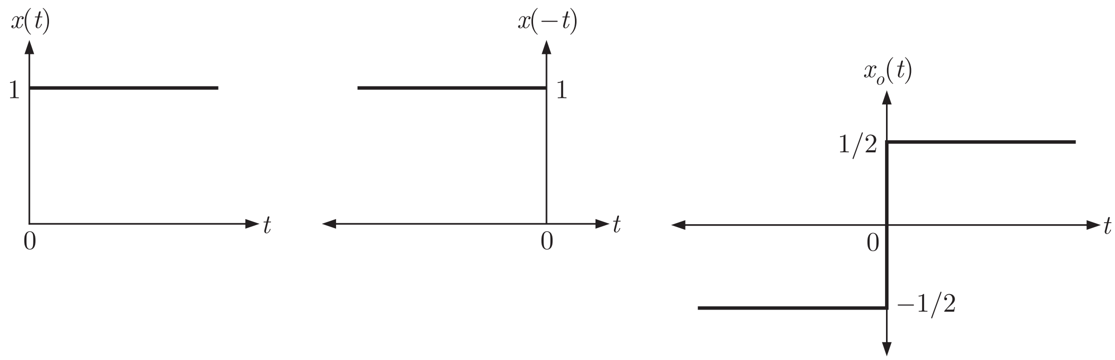
Unit step signal is given as

$$x(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

Odd part is given by

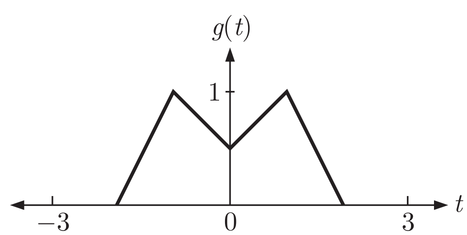
$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

This is shown graphically as below :



SOL 1.2.13 Option (D) is correct.

Shift $x(t)$ 3/4 units to the left and 3/4 units to the right and then adding both together, we get $g(t)$ as shown below :



The signal $g(t)$ is symmetrical with respect to vertical axis so odd part $g_o(t) = 0$

SOL 1.2.14 Option (D) is correct.

For an odd signal

$$x_o(-t) = -x_o(t)$$
$$x_o(t) = -x_o(-t)$$
$$x_o(0) = -x_o(-0)$$

The only number with $a = -a$ is $a = 0$ so $x_o(0) = 0$

For a signal we write

$$x(t) = x_e(t) + x_o(t)$$

For $t = 0$, $$x(0) = x_e(0) + x_o(0)$$
$$= x_e(0) + 0 = x_e(0) \qquad \text{Since } x_o(0) = 0$$

SOL 1.2.15 Option (B) is correct.

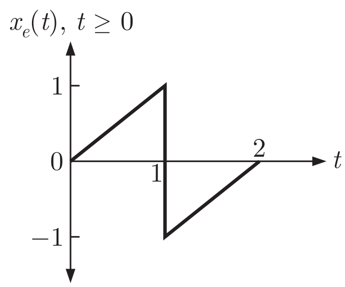
For any odd signal $x_o(-t) = -x_o(t)$. Thus the complete odd part is in option (B).

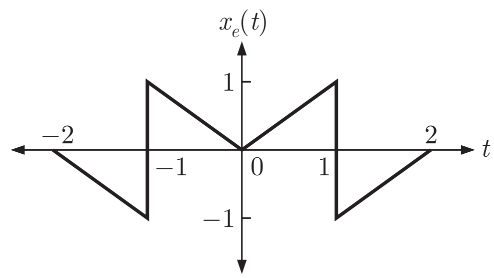
SOL 1.2.16 Option (D) is correct.

For any signal $x(t) = x_e(t) + x_o(t)$

or $x_e(t) = x(t) - x_o(t)$

Since we have $x(t)$ and $x_o(t)$ for $t \geq 0$ only, from above equation we can plot $x_e(t)$ for $t \geq 0$ as shown below.

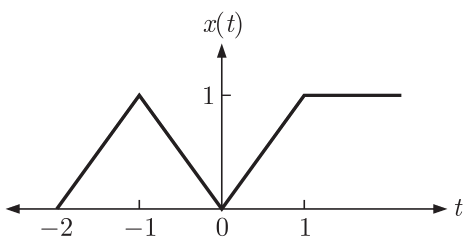




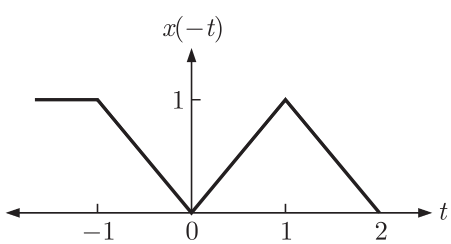
Even part of any signal is symmetric about vertical axis that is $x_e(-t) = x_e(t)$. Thus the complete even part is as shown above.

SOL 1.2.17 Option (D) is correct.

Given signal is shown below :

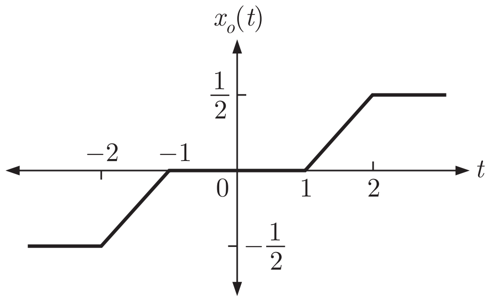


By folding the signal with respect to vertical axis



Odd part, $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

which is shown below



SOL 1.2.18 Option (B) is correct.

For signal $g_1(t)$

Energy, $$E_1 = \int_{-\infty}^{\infty} |g_1(t)|^2 dt = \int_{-2}^{2} 25\,dt = 100$$

Average Power, $$P_1 = \lim_{T\to\infty}\frac{1}{T}E_1 = 0$$

Since $g_1(t)$ has finite energy, it is an energy signal.

For signal $g_2(t)$

Energy, $$E_2 = \int_{-\infty}^{\infty} |g_2(t)|^2 dt = \infty$$

Average power, $$P_2 = \frac{1}{8}\int_{-4}^{4} |g_2(t)|^2 dt$$

$$= \frac{1}{8}\int_{-2}^{2} 25\,dt = \frac{1}{8}\times 100 = 12.5$$

The signal $g_2(t)$ has finite power, so it is a power signal.

Alternate Method :

We know that most periodic signals are usually power signals and most non-periodic signals are considered to be energy signals. $g_1(t)$ is non-periodic, so it is an energy signal. $g_2(t)$ is periodic so it is a power signal.

SOL 1.2.19 Option (B) is correct.

Energy, $$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-3}^{3} 25\,dt = 150$$

Average Power, $$P_g = \lim_{T\to\infty}\frac{1}{T}E_g = 0$$

SOL 1.2.20 Option (D) is correct.

Energy, $$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \infty$$

Average Power, $P_x = \frac{1}{8}\int_{-4}^{4}|x(t)|^2 dt$

$= \frac{1}{8}\int_{-2}^{2} 25\,dt = \frac{100}{8} = 12.5$

SOL 1.2.21 Option (D) is correct.
The signal is unbounded, therefore it is not an energy signal.

SOL 1.2.22 Option (C) is correct.

$$x(t) = 20\cos(5t)\cos(10t)\,\text{V}$$
$$= 10[\cos 15t + \cos 5t] \qquad 2\cos A\cos B = \cos(A-B) + \cos(A+B)$$
$$= 10\cos 15t + 10\cos 5t$$

Power $P_x = \frac{(10)^2}{2} + \frac{(10)^2}{2} = 100\text{ W}$

rms value $X_{rms} = \sqrt{100} = 10$ volt

SOL 1.2.23 Option (A) is correct.
Here $|x(t)| = |e^{j(2t+\pi/4)}| = 1$

Energy of the signal $E_x = \int_{-\infty}^{\infty}|x(t)|^2 dt = \int_{-\infty}^{\infty} 1\,dt = \infty$

The power of signal, $P = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2 dt$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} 1\,dt = \lim_{T\to\infty}\frac{1}{2T}(2T) = 1$$

Since $x(t)$ has finite power and infinite energy, therefore it is a power signal.

SOL 1.2.24 Option (B) is correct.

Power, $P_x = \frac{1}{T}\int_{0}^{T}|x(t)|^2 dt,$ $\qquad T \to$ Period

$$= \frac{1}{7}\int_{0}^{7}|x(t)|^2 dt$$

$$P_x = \frac{1}{7}\left[\int_{0}^{2}(0)^2 dt + \int_{2}^{5}(4)^2 dt + \int_{5}^{7}(2)^2 dt\right]$$

$$= \frac{1}{7}[0 + (16\times 3) + (4\times 2)]$$

$$= 8\text{ unit}$$

SOL 1.2.25 Option (A) is correct.
Energy E_x of signal $x(t)$ is given as

$$E_x = \int_{-3}^{3}|x(t)|^2 dt = 12\text{ units}$$

Energy of $2x(t)$,

$$E_1 = (2)^2 \times E_x = 4\times 12 = 48\text{ unit}$$

Let, $x_2(t) = x(3t)$

So, $x_2(t)$ is defined over the range $-1 \le t \le 1$

Energy $$E_2 = \int_{-1}^{1} |x_2(t)|^2 dt = \int_{-1}^{1} |x(3t)|^2 dt$$

Let $3t = \alpha \longrightarrow dt = \frac{1}{3} d\alpha$

So $$E_2 = \frac{1}{3}\int_{-3}^{3} |x(\alpha)|^2 d\alpha = \frac{1}{3} \times E_x = 4 \text{ unit}$$

Energy of $x(t-4)$ is same as $x(t)$.

Energy of $2x(2t)$

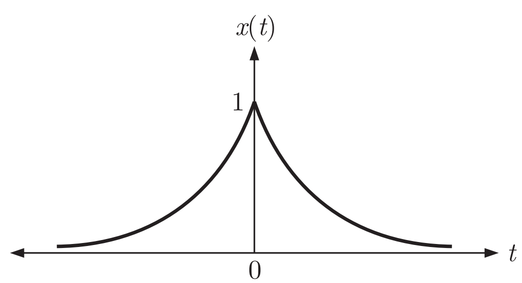
$$E_4 = (2)^2 \times \frac{1}{2} E_x = 24 \text{ unit}$$

SOL 1.2.26 Option (B) is correct.

$$x(t) = e^{-|t|},$$
$$x(-t) = e^{-|-t|} = e^{-|t|} = x(t)$$

Since $x(t) = x(-t)$, it is an even signal.

Signal $x(t)$ is bounded, so it is has some finite energy.



SOL 1.2.27 Option (A) is correct.

$y(t)$ is multiplication of $x_1(t)$ and $x_2(t)$.

For interval $0 \le t \le 1$, $x_1(t) = t,\ x_2(t) = 1$

so, $y(t) = x_1(t)\,x_2(t) = t$

For $1 \le t \le 2$, $x_1(t) = 1,\ x_2(t) = 0.5$

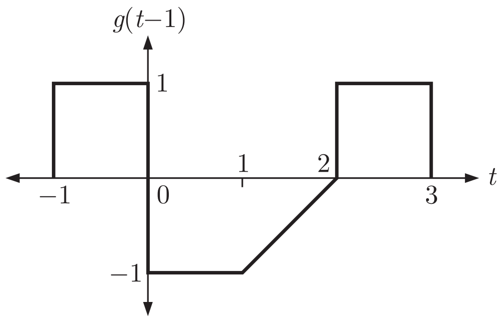
$y(t) = x_1(t)\,x_2(t) = 0.5$

For $2 \le t \le 3$, $x_1(t) = 0.5,\ x_2(t) = 1.5$

$y(t) = x_1(t)\,x_2(t) = 0.75$

SOL 1.2.28 Option (C) is correct.

Shift $g(t)$ to the right by one time unit to obtain $g(t-1)$ as shown below :



For $-1 \le t \le 0$, $\quad f(t) = -t-1,\ g(t-1) = 1$

So, $\quad x(t) = -t-1$

For $0 \le t \le 1$, $\quad f(t) = t,\ g(t-1) = -1$

So, $\quad x(t) = -t$

For $1 \le t \le 2$, $\quad f(t) = 1,\ g(t-1) = t-2$

So, $\quad x(t) = t-2$

For $2 \le t \le 3$ $\quad f(t) = -t+3,\ g(t-1) = 1$

So, $\quad x(t) = -t+3$

SOL 1.2.29 Option (D) is correct.

Put $t = 2\alpha$,

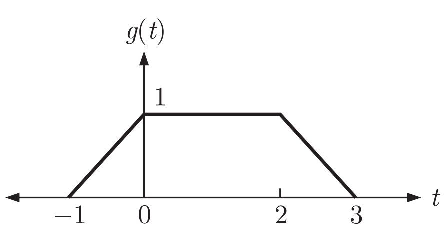
$$g(2\alpha) = \begin{cases} 2\alpha+1, & -1 \le 2\alpha \le 0 \\ 1, & 0 \le 2\alpha < 2 \\ 0, & \text{else where} \end{cases}$$

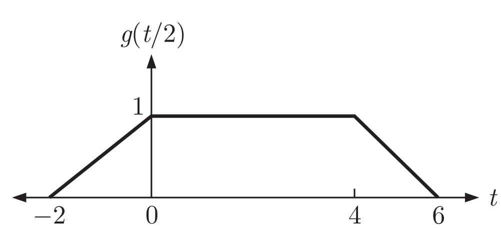
Changing the variable($\alpha \to t$)

$$g(2t) = \begin{cases} 2t+1, & -\frac{1}{2} \le t \le 0 \\ 1, & 0 \le t < 1 \\ 0, & \text{else where} \end{cases}$$

SOL 1.2.30 Option (C) is correct.

The waveform for signal $g(t)$ and $g(t/2)$ are drawn as below.

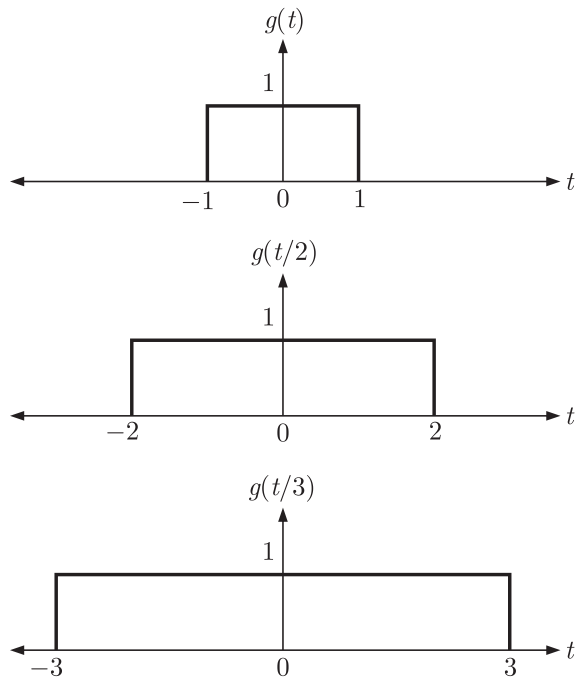




Signal $g(t/2)$ is obtained by expanding the $g(t)$ by a factor of 2 in the time domain.

SOL 1.2.31 Option (C) is correct.

The signal $g(t)$ and its expanded signal by factor of 2 and 3 is shown below :

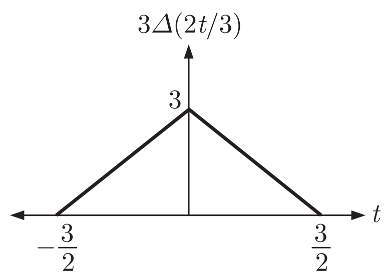


By adding all three, we get

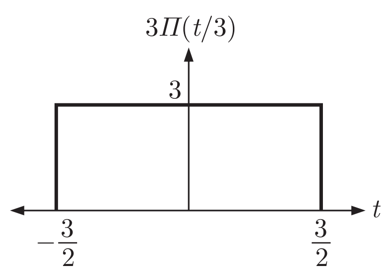
$$f(t) = g(t) + g(t/2) + g(t/3)$$

SOL 1.2.32 Option (B) is correct.

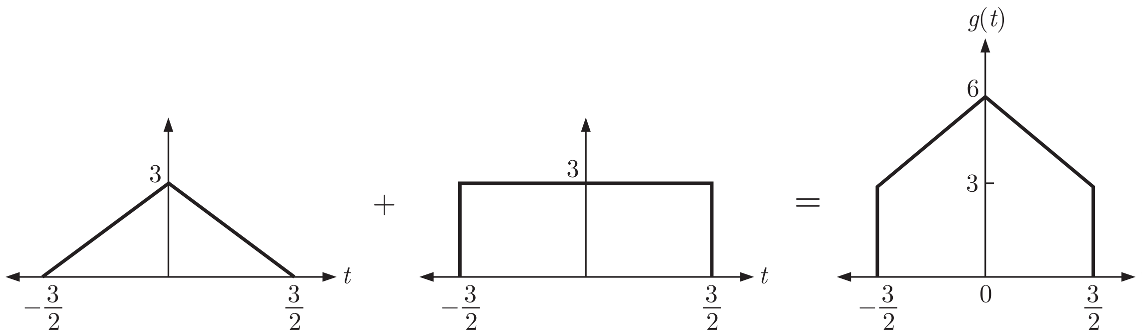
$3\Delta(2t/3)$ is obtained by expanding $\Delta(t)$ with a factor of $3/2$ and scaling amplitude by a factor of 3.



Similarly, to get $3\Pi(t/3)$, expand $\Pi(t)$ by a factor of 3 and amplitude scale by 3



Now adding both signal we get



SOL 1.2.33 Option (A) is correct.

Energy of a signal $x(t)$, $E_x = \int_{-\infty}^{\infty} |x(t)|^2 t$

Now let the signal is time compressed by a factor of a

$$y(t) = x(at)$$

Energy of $y(t)$ $$E_y = \int_{-\infty}^{\infty} |x(at)|^2 dt$$

$$at = \alpha \Rightarrow dt = \frac{1}{a} d\alpha$$

$$E_y = \frac{1}{a}\int_{-\infty}^{\infty} |x(\alpha)|^2 d\alpha = \frac{1}{a} E_x$$

So due to time compression energy reduces.

SOL 1.2.34 Option (B) is correct.

To get $g(t+2)$ shift $g(t)$ to the left by 2 time units. The signal is advanced by 2 time units.

SOL 1.2.35 Option (D) is correct.

The signal $y(t)$ is the time delayed version of $x(t)$ i.e $y(t) = x(t-2)$

SOL 1.2.36 Option (A) is correct.

The delayed version of $x(t)$,

$$y(t) = x(t-3)$$

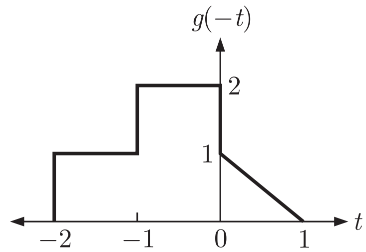
can be obtained directly by shifting $x(t)$ to the right by 3 sec.

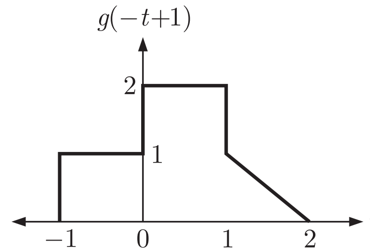
SOL 1.2.37 Option (C) is correct.

The time delayed signal $g(t-2)$ can be obtained by shifting $g(t)$ to the right by 2 time units.

SOL 1.2.38 Option (C) is correct.

First time reverse the signal $g(t)$ to get $g(-t)$ and then shift $g(-t)$, toward right to get $g(-t+1)$ as shown in figure





SOL 1.2.39 Option (A) is correct.

We have
$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

Energy of signal $x(at-b)$,
$$E_2 = \int_{-\infty}^{\infty} |x(at-b)|^2\, dt$$

Put $at - b = \alpha \longrightarrow dt = \frac{1}{a} d\alpha$

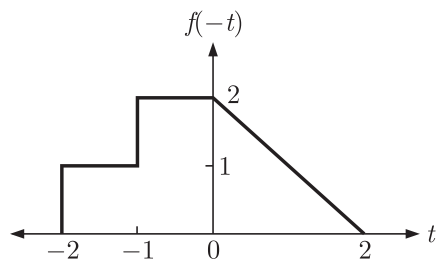
So
$$E_2 = \frac{1}{a}\int_{-\infty}^{\infty} |x(\alpha)|^2\, d\alpha = \frac{1}{a}E_x$$

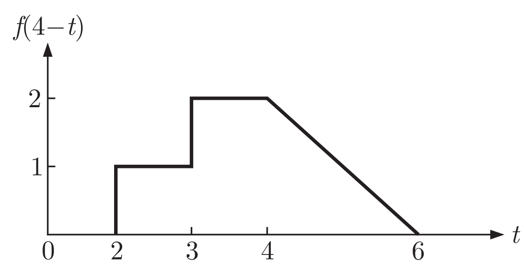
SOL 1.2.40 Option (C) is correct.

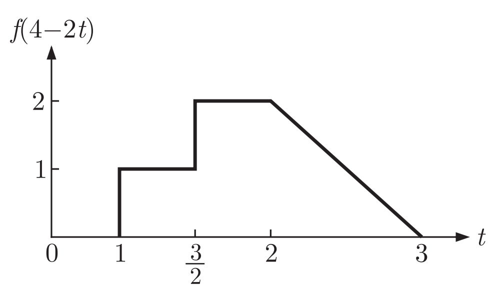
The sequence of transformation is

$$f(t) \xrightarrow[\text{time revenal}]{t \to -t} f(-t) \xrightarrow[\text{time shift}]{t \to t-4} f(4-t) \xrightarrow[\text{time scaling}]{t \to 2t} f(4-2t)$$

This can be performed in following steps





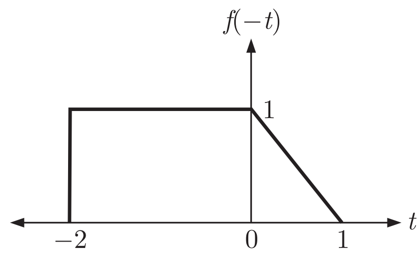


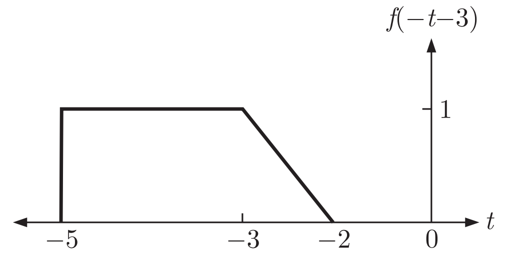
Alternate Method : As given in methodology of section 1.4, we can also follow the other sequence of operation which is given as

$$f(t) \xrightarrow[\text{time shift}]{t \to t+4} f(t+4) \xrightarrow[\text{time scaling}]{t \to 2t} f(2t+4) \xrightarrow[\text{time revenal}]{t \to -t} f(-2t+4)$$

SOL 1.2.41 Option (C) is correct.

First we obtain time reversal signal $f(-t)$ by taking mirror image of $f(t)$ along the vertical axis. Then by shifting $f(-t)$ to the left by 3 units we get $f(-t-3)$.





SOL 1.2.42 Option (C) is correct.

We can see that $y(2) = x(0)$ [origin is shifted at 2]

so $2a + b = 0$ ...(i)

Similarly $y(8/3) = x(2)$

So $\frac{8}{3}a + b = 2$ ...(ii)

From eq (i) and (ii) $a = 3,\ b = -6$

SOL 1.2.43 Option (C) is correct.

From the graph we can write $x_2(t) = x_1(3t-6) = x_1[3(t-2)]$. So $x_2(t)$, can be obtained by compressing $x_1(t)$ by a factor of 3 and then delaying by 2 time units.

Alternate Method :

As given in methodology of section 1.4, $x_2(t)$ can be obtained by shifting $x_1(t)$ by 6 time units to the right and then by scaling(compressing) it with a factor of 3. This is not given in any of the four options.

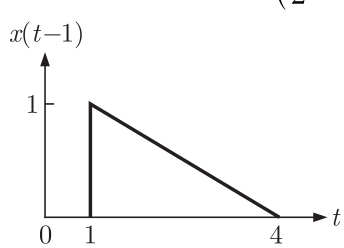
SOL 1.2.44 Option (B) is correct.

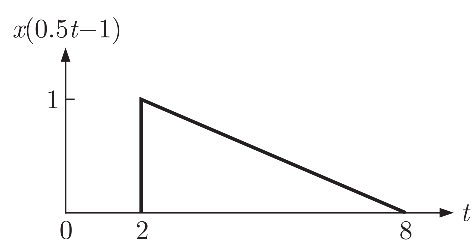
$$x_1(t) = x[0.5(t-2)]$$

or

$$x_1(t) = x(0.5t-1)$$

First shift $x(t)$ to right by one unit to get $x(t-1)$. Then, expand $x(t-1)$ by a factor of 2 to get $x\left(\frac{t}{2}-1\right)$ or $x(0.5t-1)$





If we change sequence of transformation by first doing scaling then shifting we get

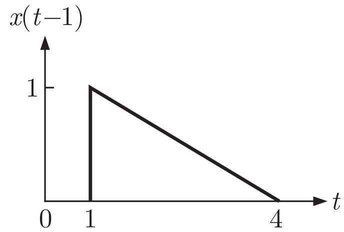
$$x(t) \xrightarrow[\text{time scaling}]{t \to 0.5t} x(0.5t) \xrightarrow[\text{time shifting}]{t \to t-1} x[0.5(t-1)] \neq x[0.5t-1]$$

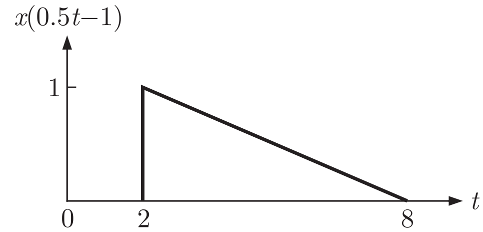
Hence (B) is correct option.

SOL 1.2.45 Option (C) is correct.

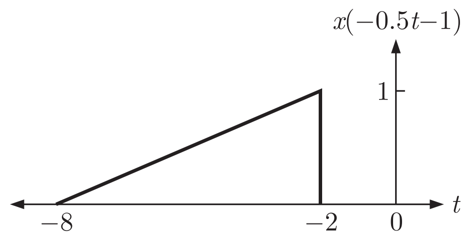
$$x_2(t) = x(-0.5t - 1)$$

First shift $x(t)$ to the right by 1 unit, we get $x(t-1)$. Then, expand $x(t-1)$ by a factor of 2 to get $x(t/2 - 1)$





Now fold signal $x(0.5t - 1)$ about the vertical axis to get $x(-0.5t - 1)$



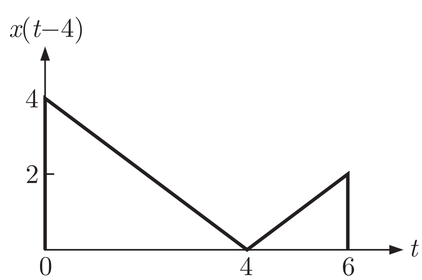
If we change the order of transformation we get

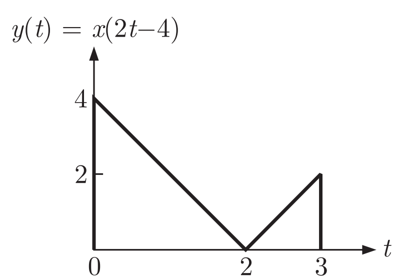
$$x(t) \xrightarrow[\text{Time scaling}]{t \to 0.5t} x(0.5t) \xrightarrow[\text{Time shifting}]{t \to t-1} x[0.5\,(t-1)] \xrightarrow[\text{Time reversal}]{t \to -t} x[-0.5t - 0.5] \neq x[-0.5t - 1]$$

Time scaling and time reversal are commutative, so we may change their order.

SOL 1.2.46 Option (B) is correct.

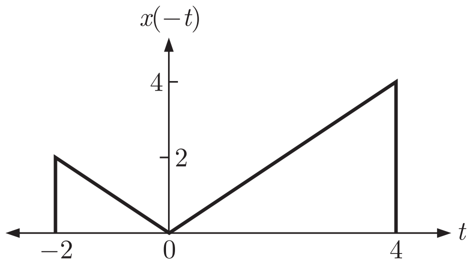
In multiple transformation, we first do shifting then time scaling. From $y(t)$, we can see that $x(t)$ is shifted to right by 4 time units to get $x(t-4)$. Then it is time expanded by a factor of 2 to get $x(2t-4)$

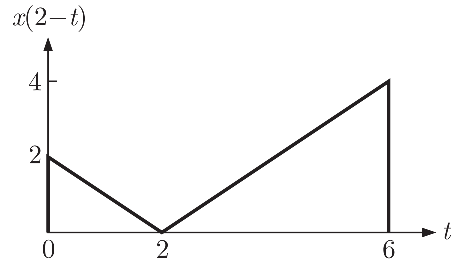




SOL 1.2.47 Option (C) is correct.

First fold $x(t)$, with respect to vertical axis. Then shift $x(-t)$ toward right by 2 time units, to get $x(-t+2)$





SOL 1.2.48 Option (C) is correct.

From the graphs, we can see that signal has no time shift (because origin is not shifted), so $t_0 = 0$. Signal $x(t)$ is magnitude scaled by a factor of -2.

Since, $y(t)$ has half duration of $x(t)$, so it is time compressed by a factor of 2.

$$W = \frac{1}{2}$$

$$y(t) = -2x\left(\frac{t}{\frac{1}{2}}\right) = -2x(2t)$$

SOL 1.2.49 Option (B) is correct.

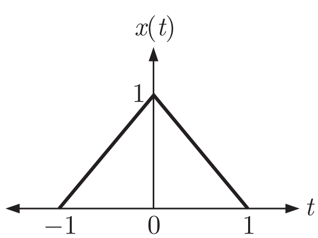
The sequence of transformation

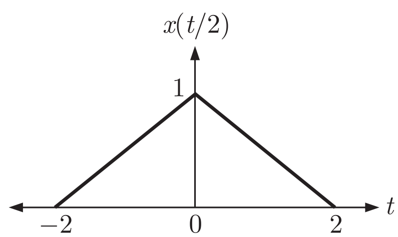
$$x(t) \xrightarrow[\text{time scaling}]{t \to t/2} x\left(\frac{t}{2}\right) \xrightarrow[\text{time shifting}]{t \to t-1} x\left(\frac{t-1}{2}\right) \xrightarrow[\text{amplitude scaling}]{-6} -6x\left(\frac{t-1}{2}\right)$$

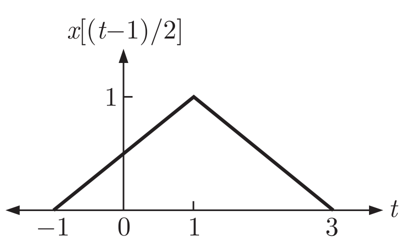
If we change the order of transformation.

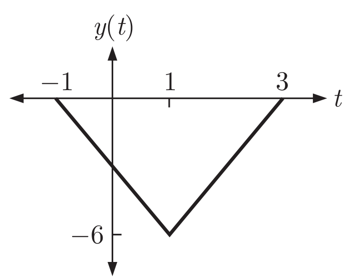
$$x(t) \xrightarrow{t \to t-1} x(t-1) \xrightarrow{t \to t/2} x\left(\frac{t}{2} - 1\right) \neq x\left(\frac{t-1}{2}\right)$$

Graphically







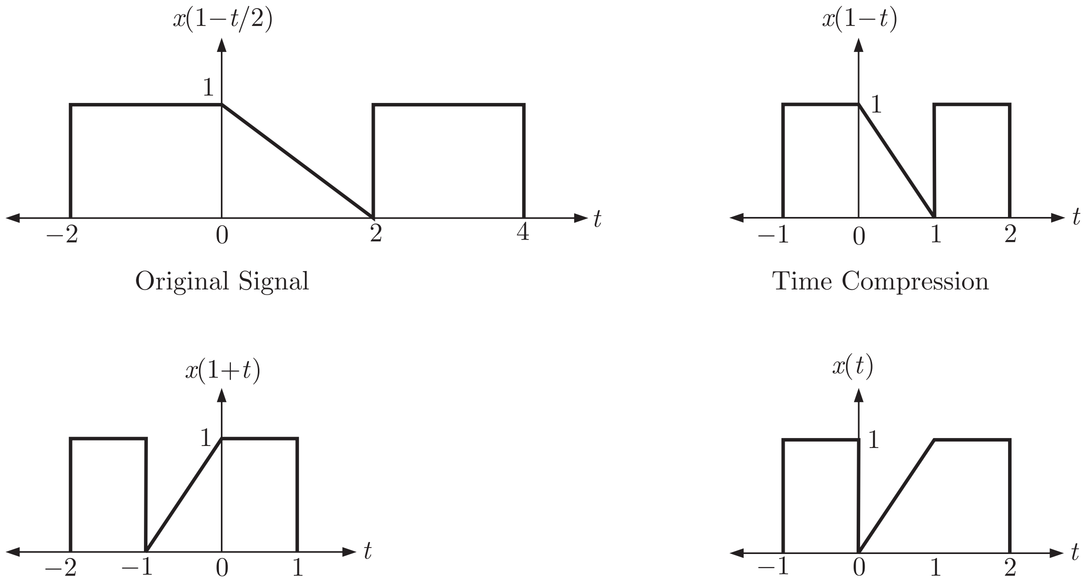


SOL 1.2.50 Option (C) is correct.

We can perform following sequence of transformation.

$$x\left(1 - \tfrac{t}{2}\right) \xrightarrow[\text{time compression}]{t \to 2t} x(1-t) \xrightarrow[\text{folding}]{t \to -t} x(t+1) \xrightarrow[\text{time shifting}]{t \to t-1} x(t)$$

Graphically it is obtained as



Original Signal

Time Compression

Time Reflection

Time Shifting

SOL 1.2.51 Option (C) is correct.

$$x(t) = \int_{-\infty}^{\infty} e^{-t}\delta(2t-2)\,dt = \int_{-\infty}^{\infty} e^{-t}\delta[2(t-1)]\,dt \qquad \delta[2(t-1)] = \frac{1}{2}\delta(t-1)$$

$$= \int_{-\infty}^{\infty} e^{-t}\frac{1}{2}\delta(t-1)\,dt = \frac{1}{2}\int_{-\infty}^{\infty} e^{-t}\delta(t-1)\,dt$$

$$= \frac{1}{2}e^{-t}\Big|_{at\;\;t=1} \qquad \int_{-\infty}^{\infty} f(t)\,\delta(t-t_0) = f(t_0)$$

$$= \frac{1}{2e}$$

SOL 1.2.52 Option (C) is correct.

From the scaling property of impulse function we can see that

$$\delta[a(t-t_0)] = \frac{1}{|a|}\delta(t-t_0)$$

SOL 1.2.53 Option (C) is correct.

$$g(t) = 6\delta(3t+9) = 6\delta[3(t+3)]$$

$$= \frac{6}{3}\delta(t+3) \qquad \delta[a(t+b)] = \frac{1}{a}\delta(t+b)$$

$$= 2\delta(t+3)$$

So, $g(t)$ is an impulse with magnitude of 2 unit at $t = -3$.

SOL 1.2.54 Option (B) is correct.

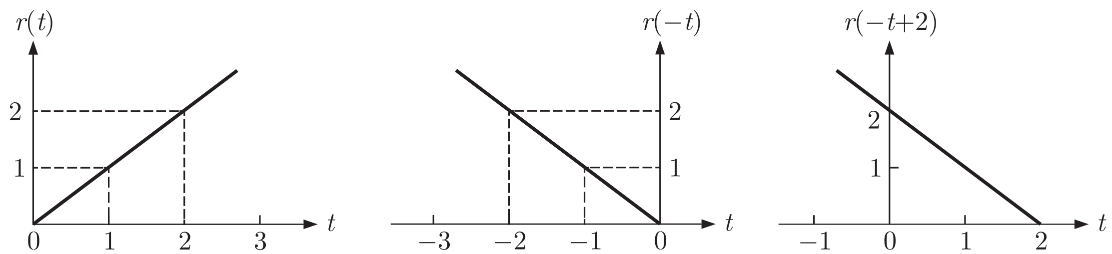
Here we can apply the shifting property of impulse function as below

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$$

Thus $\quad x(t) = \int_{-\infty}^{\infty} \delta(t+5)\cos(\pi t)\,dt = \cos(\pi t)\big|_{t=-5} = \cos(-5\pi) = -1$

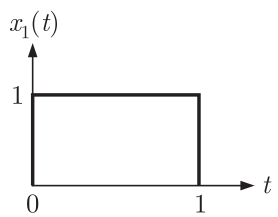
SOL 1.2.55 Option (C) is correct.

First, fold the signal about $t=0$ to get $r(-t)$ and then shift $r(-t)$ toward right to get $r(-t+2)$ as shown below



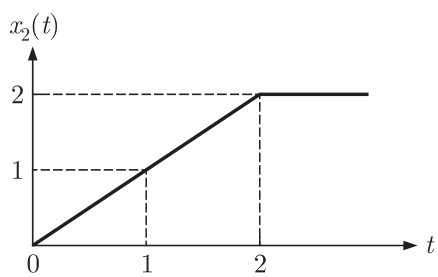
SOL 1.2.56 Option (B) is correct.

The signal $x_1(t)$ is shown below



$$E_1 = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_0^1 1\,dt = 1\,\text{unit}$$

The signal $x_2(t)$ is shown below



$$E_2 = \int_{-\infty}^{\infty} |x_2(t)|^2\,dt$$

$$= \int_0^2 t^2\,dt + \int_2^{\infty} 4\,dt = \infty \qquad x_2 \text{ is unbounded}$$

Energy of $x_3(t)$

$$E_3 = \int_{-\infty}^{\infty} |x_3(t)|^2\,dt = \int_0^{\infty} (1+e^{-6t})^2\,dt$$

$$= \int_0^{\infty} (1+e^{-12t}+2e^{-6t})\,dt = \infty \quad (x_3 \text{ is unbounded})$$

So, only $x_1(t)$ has finite energy.

SOL 1.2.57 Option (B) is correct.

$$x(t) = u(t+2) - 2u(t) + u(t-2)$$

To draw $x(t)$, we observe change in amplitude at different instants.

1. First at $t=-2$, $x(t)$ steps up with amplitude 1.
2. At $t=0$, another step is added with amplitude of -2. So, the net amplitude

becomes $[1+(-2)]=-1$.

3. Similarly at $t=2$, a step with amplitude 1 is added which causes net amplitude $(-1+1)=0$.

SOL 1.2.58 Option (C) is correct.

To sketch $x(t)$, we observe change in amplitude of step signals at different instants of time.

1. At $t=-3$, a step with magnitude -1 is added.
2. At $t=-1$, another step of magnitude $+2$ is added which causes net magnitude $(2-1)=1$.
3. At $t=1$, a step of magnitude -2 is added so net magnitude becomes $(1-2)=-1$.
4. At $t=3$, a step with magnitude 1 is added, Now magnitude is $(-1+1)=0$.

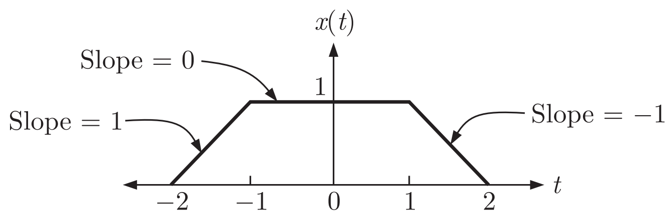
SOL 1.2.59 Option (B) is correct.

$$x(t)=r(t+2)-r(t+1)-r(t-1)+r(t-2)$$

To sketch $x(t)$, we observe change in slope at different instants of time.

1. At $t=-2$, a ramp with slope of 1 is added.
2. At $t=-1$, a ramp with slope of -1 is added, so net slope becomes $(-1+1)=0$
3. Similarly, at $t=1$, a ramp of slope -1 is added with causes net slope $(-1+0)=-1$
4. Again, at $t=2$ a ramp of slope 1 is added and the net slope becomes zero.

The correct sketch is



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SOLUTIONS 1.3

SOL 1.3.1 Option (A) is correct.

Period of $x(t)$, $$T = \frac{2\pi}{\omega} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ sec}$$

SOL 1.3.2 Option (C) is correct.

Period of $\sin 5t$, $$T_1 = \frac{2\pi}{5}$$

Period of $\cos 7t$, $$T_2 = \frac{2\pi}{7}$$

Period of $x(t)$, $$T = \text{LCM}\left(\frac{2\pi}{5}, \frac{2\pi}{7}\right) = 2\pi$$

SOL 1.3.3 Option (D) is correct.

Signal $x(t)$ is not periodic because of the term $5t$ which is aperiodic in nature.

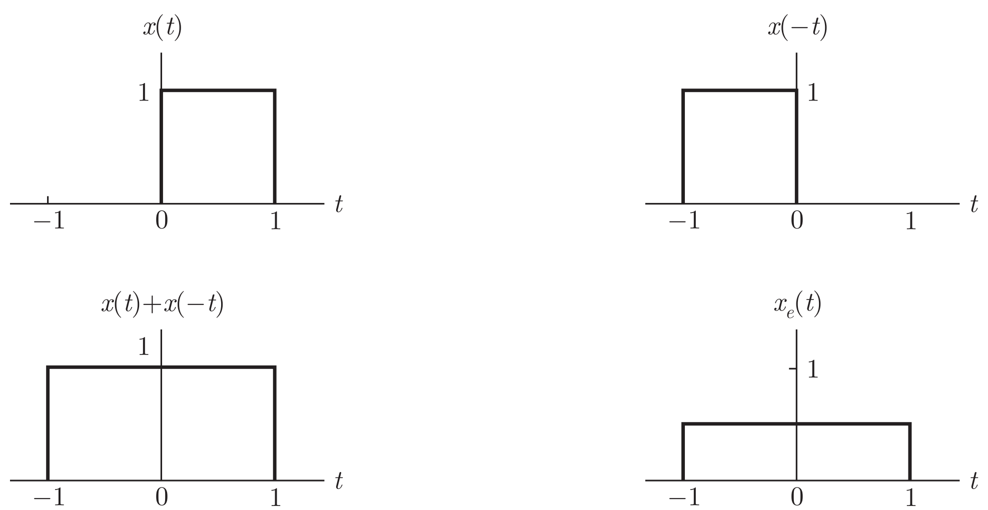
SOL 1.3.4 Option (D) is correct.

Not periodic because least common multiple of periods of $\sin 3t$ and $\sin\sqrt{t}$ is infinite.

SOL 1.3.5 Option (A) is correct.

Even part of $x(t)$, $$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

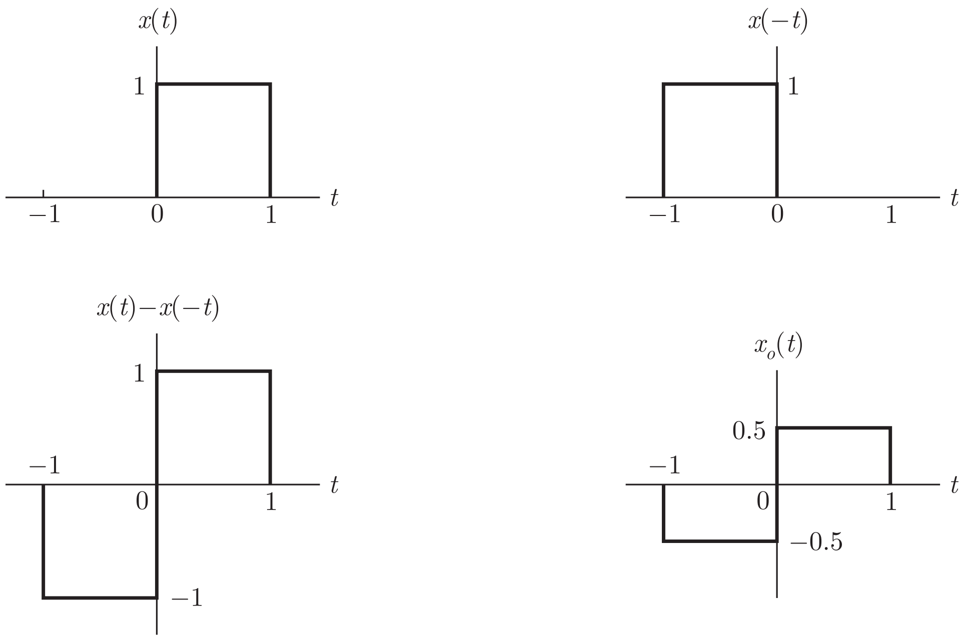
This can be obtained graphically in following steps :



SOL 1.3.6 Option (C) is correct.

Odd part of $x(t)$, $$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

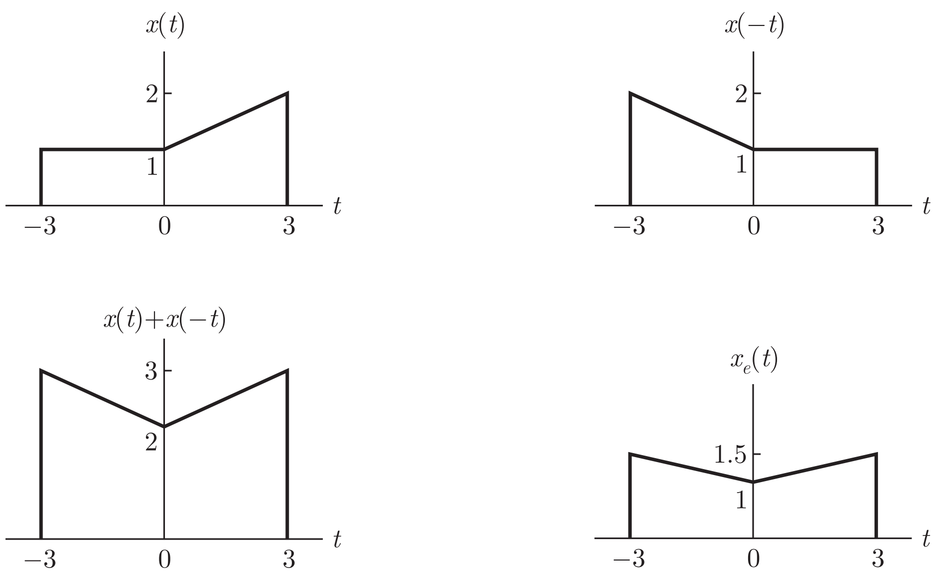
This can be obtained graphically in following steps :



SOL 1.3.7 Option (A) is correct.

Even part of $x(t)$, $$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

Signal $x_e(t)$ is obtained as follows :



SOL 1.3.8 Option (C) is correct.

This is energy signal because

$$E_\infty = \int_{-\infty}^{\infty} x(t)\,dt < \infty = \int_{-\infty}^{\infty} e^{-4t} u(t)\,dt = \int_0^{\infty} e^{-4t}\,dt = \frac{1}{4}$$

SOL 1.3.9 Option (A) is correct.

Energy of signal $x(t)$, $E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt$

$$= \int_{-\infty}^{\infty} (1)\,dt = \infty \qquad \text{Since } |x(t)| = 1$$

Energy of $x(t)$ is infinite, therefore this is a power signal not an energy signal.

Power of $x(t)$, $P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt = 1$

SOL 1.3.10 Option (A) is correct.

Energy of signal $x(t)$, $E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-\pi/\omega}^{\pi/\omega} \frac{1}{4}(\cos\omega t + 1)^2\,dt$

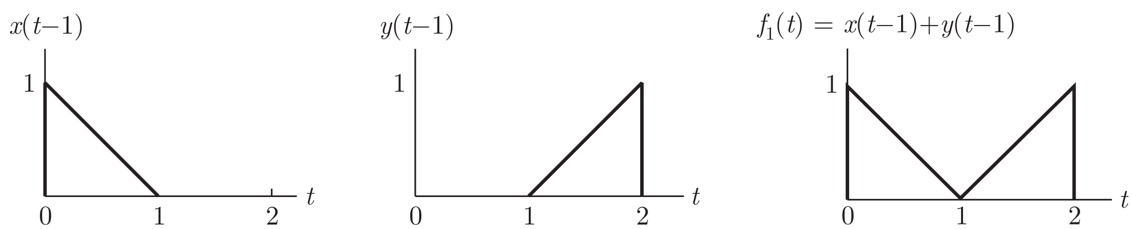
$$= \frac{2}{4} \int_0^{\pi/\omega} (\cos^2\omega t + 2\cos\omega t + 1)\,dt$$

$$= \frac{1}{2} \int_0^{\pi/\omega} \left(\frac{1}{2}\cos 2\omega t + \frac{1}{2} + 2\cos\omega t + 1\right) dt$$

$$= \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{\pi}{\omega}\right) = \frac{3\pi}{4\omega}$$

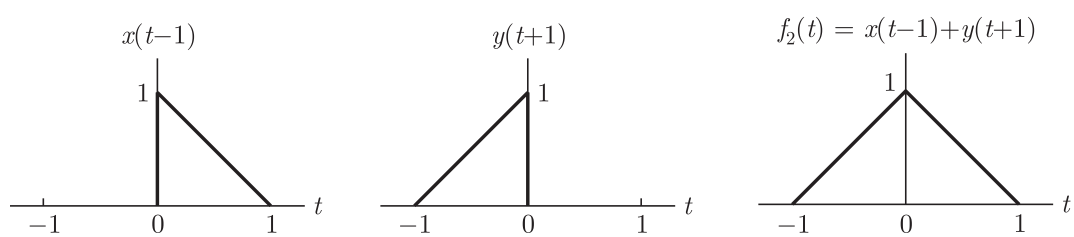
SOL 1.3.11 Option (B) is correct.

First we shift $x(t)$ and $y(t)$ to the right by 1 unit, to get $x(t-1)$ and $y(t-1)$ respectively. Now by adding $x(t-1)$ and $y(t-1)$, we get $f_1(t)$ as shown below



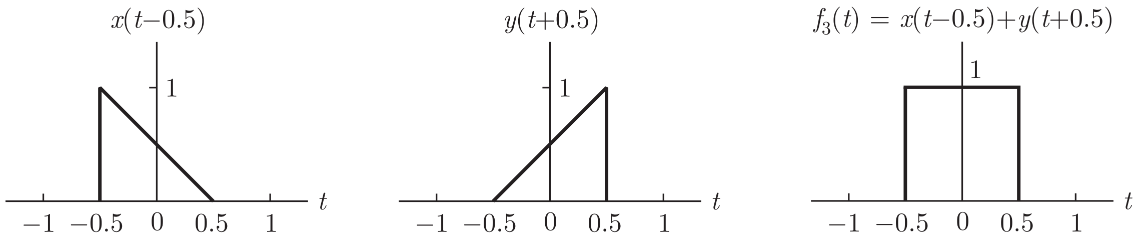
SOL 1.3.12 Option (A) is correct.

First we shift $x(t)$ to the right by 1 unit to get $x(t-1)$ and $y(t)$ to the left by 1 unit to get $y(t+1)$. Now, adding $x(t-1)$ and $y(t+1)$ we will get $f_2(t)$ as shown below



SOL 1.3.13 Option (A) is correct.

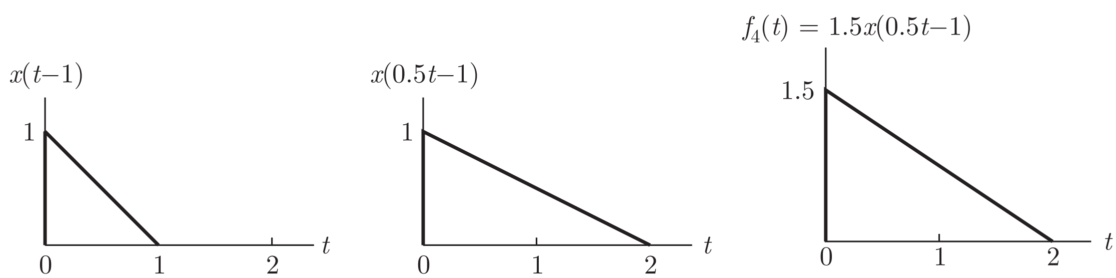
First we shift $x(t)$ to the right by 0.5 unit, and $y(t)$ to the left by 0.5 unit to get $x(t-0.5)$ and $y(t+0.5)$ respectively. Now, adding $x(t-0.5)$ and $y(t+0.5)$ we will get $f_3(t)$ as shown below



Sol. 5.1.31

SOL 1.3.14 Option (D) is correct.

$f_4(t)$ can be obtained by performing multiple operation on $x(t)$. First delay $x(t)$ by 1 unit, we get $x(t-1)$. Now, time expand $x(t-1)$ by a factor of 2, we get $x(t/2-1)$ or $x(0.5t-1)$. In last step, $f_4(t)$ can is obtained by multiplying $x(0.5t-1)$ with a constant 1.5. Graphically, these steps are performed as shown below :



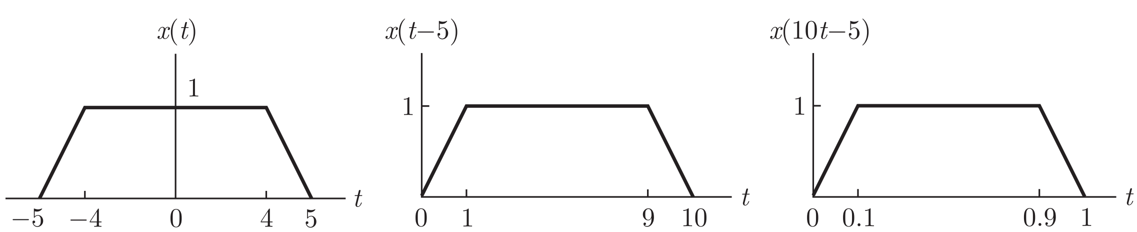
SOL 1.3.15 Option (C) is correct.

$$y(t) = x(10t-5)$$

The sequence of transformation is

$$x(t) \xrightarrow[\text{time shift}]{t \to t-5} x(t-4) \xrightarrow[\text{time scaling}]{t \to 10t} x(10t-5)$$

This can be performed in following steps



SOL 1.3.16 Option (D) is correct.

Multiplication of independent variable t by 5 will bring compression on time scale. It may be checked by $x(5 \times 0.8) = x(4)$.

SOL 1.3.17 Option (A) is correct.

Division of independent variable t by 5 will bring expansion on time scale. It may be checked by

$$y(20) = x\left(\frac{20}{5}\right) = x(4)$$

SOL 1.3.18 Option (C) is correct.

Mathematically, the function $x(t)$ can be defined as

$$x(t) = \begin{cases} t+5, & \text{for } -5 < t < -4 \\ -t+5, & \text{for } 4 < t < 5 \\ 1, & \text{for } -4 < t < 4 \end{cases}$$

$$y(t) = \frac{dx(t)}{dt} = \begin{cases} 1, & \text{for } -5 < t < -4 \\ -1, & \text{for } 4 < t < 5 \\ 0, & \text{for } -4 < t < 4 \end{cases}$$

Energy of $y(t)$ is calculated as

$$E_y = \int_{-\infty}^{\infty} |y(t)|^2 dt = \int_{-5}^{-4} (1)^2 dt + \int_{4}^{5} (-1)^2 dt = 2$$

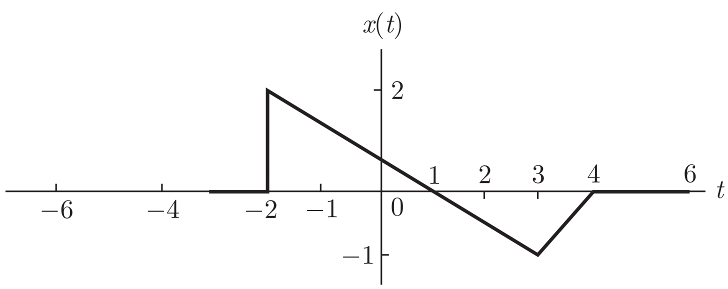
SOL 1.3.19 Option (D) is correct.

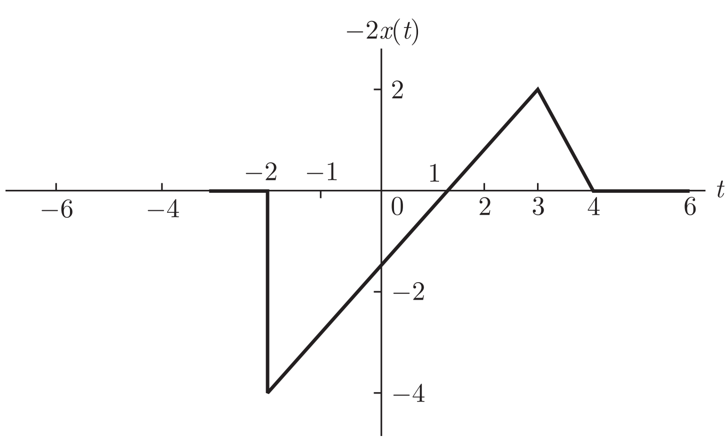
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\int_0^5 x^2(t)\,dt$$

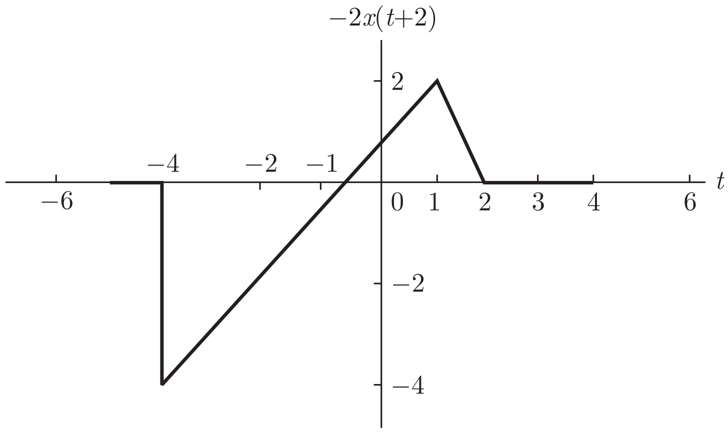
$$= 2\int_0^4 (1)^1 dt + 2\int_4^5 (5-t)^2 dt = 8 + \frac{2}{3} = \frac{36}{3}$$

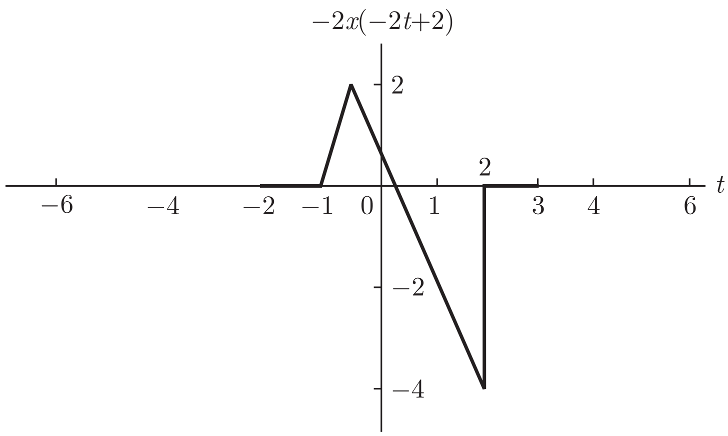
SOL 1.3.20 Option (C) is correct.

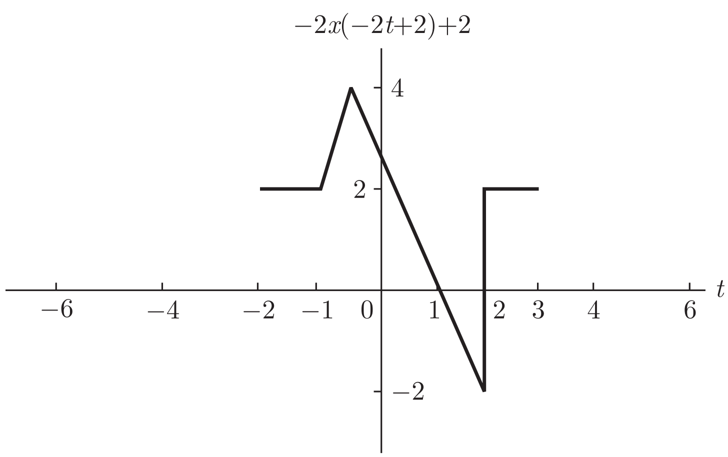
The transformation of $x(t)$ to $y(t)$ is shown as below











SOL 1.3.21 Option (A) is correct.

For an impulse function we have

$$\int_{-\infty}^{\infty} \delta(t-a)\,dt = 1, \text{ for } t=a \quad \text{otherwise } 0.$$

so, $\int_{-1}^{8}[\delta(t+3) - 2\delta(4t)]\,\delta t = \int_{-1}^{8}\delta(t+3)\,dt - 2\int_{-1}^{8}\delta(4t)\,dt$

$$= 0 - 2\int_{-1}^{8}\delta(4t) \qquad \int_{-\infty}^{\infty}\delta(t-a)\,dt = 1, \text{ for } t = a$$

$$= -\frac{2}{4}\int_{-1}^{8}\delta(t) = -\frac{1}{2} \qquad \text{since } \delta(at) = \frac{1}{a}\delta(t)$$

$\int_{-1}^{8}\delta(t+3)\,dt = 0$ because $t = -3$ does not exist in the given interval $(-1 < t < 8)$.

SOL 1.3.22 Option (C) is correct.

$$x(t) = 2\delta(2t) + 6\delta[3(t-2)]$$

$$= \frac{2}{2}\delta(t) + \frac{6}{3}\delta(t-2) \qquad \text{since } \delta a(t-t_0) = \frac{1}{a}\delta(t-t_0)$$

$$= \delta(t) + 2\delta(t-2)$$

SOL 1.3.23 Option (A) is correct.

From the shifting property of impulse function, we know that

$$\int_{-\infty}^{\infty}x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

So,

$$y(\tau) = \int_{-\infty}^{\infty}x(\tau)\,[\delta(\tau-2) + \delta(\tau+2)]\,d\tau$$

$$= \int_{-\infty}^{\infty}x(\tau)\,[\delta(\tau-2)]\,d\tau + \int_{-\infty}^{\infty}x(\tau)\,[\delta(\tau+2)]\,d\tau$$

$$= x(2) + x(-2)$$

SOL 1.3.24 Option (D) is correct.

Substituting $at = u \Rightarrow dt = \frac{1}{a}du$, we get

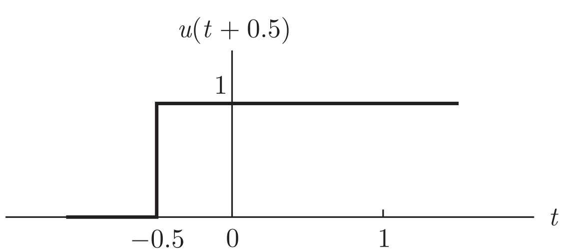
$$\int_{-\infty}^{\infty}\delta(at-b)\sin^2(t-4)\,dt = \int_{-\infty}^{\infty}\delta(u-b)\sin^2\left(\frac{u}{a}-4\right)\frac{du}{a}$$

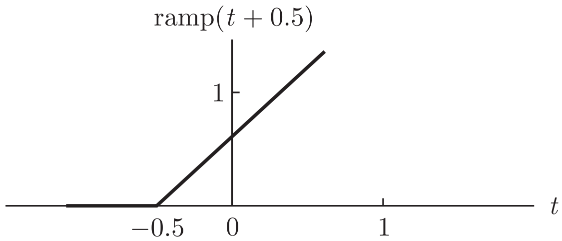
$$= \frac{1}{a}\int_{-\infty}^{\infty}\delta(u-b)\sin^2\left(\frac{u}{a}-4\right)du$$

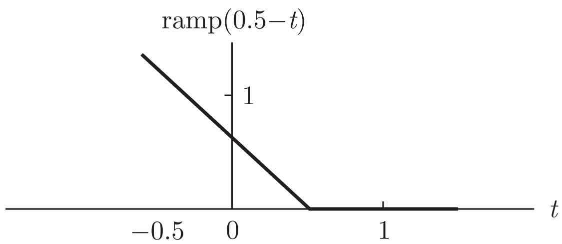
$$= \frac{\sin^2\left(\frac{b}{a}-4\right)}{a} \qquad \text{since } \int_{-\infty}^{\infty}x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

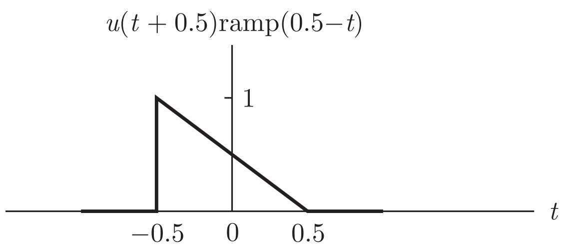
SOL 1.3.25 Option (C) is correct.

$x(t)$ is obtained in following steps :



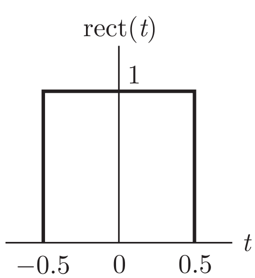


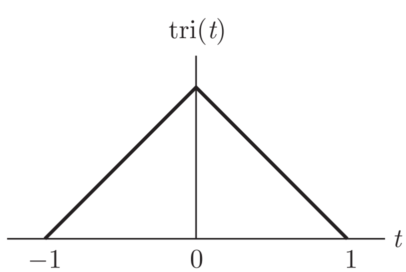


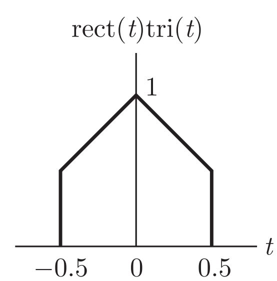


SOL 1.3.26 Option (B) is correct.

All signal are as shown below







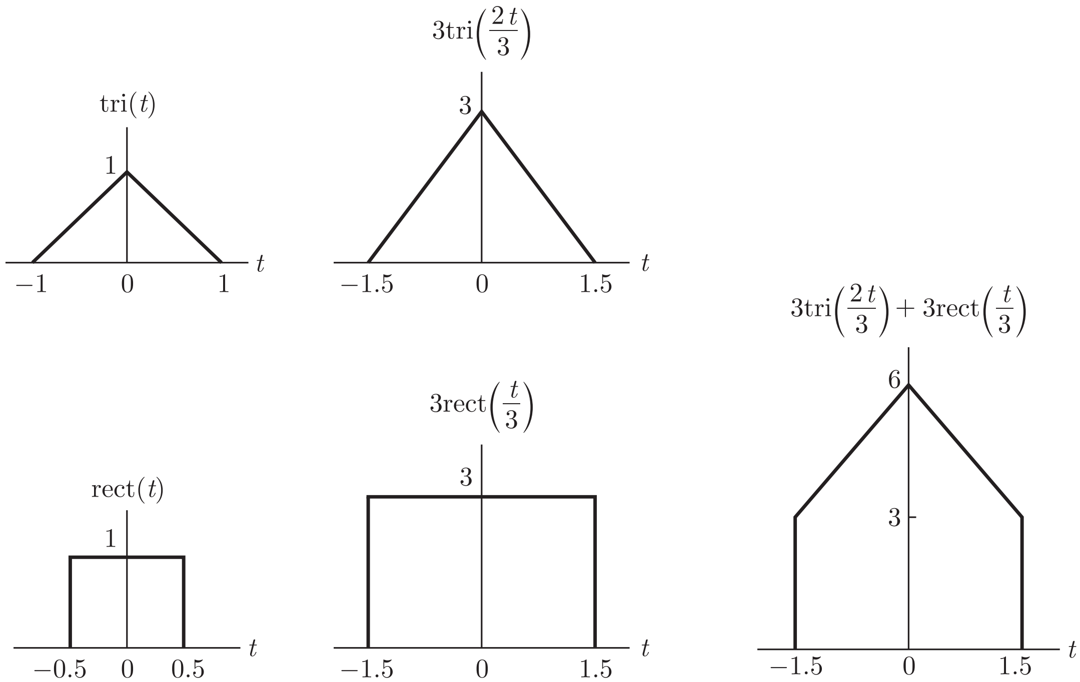
SOL 1.3.27 Option (C) is correct.

$$x(t) = 4\,\mathrm{tri}\,(t) = 4\,(1 - |\,t\,|) \quad 0 < |\,t\,| < 1$$

$$x\left(\frac{1}{2}\right) = 4\left(1 - \left|\,\frac{1}{2}\,\right|\right) = 2$$

SOL 1.3.28 Option (D) is correct.

Figure is as shown below



SOL 1.3.29 Option (B) is correct.

This is triangle with the same height as $(\frac{1}{\alpha})\,\text{tri}\,(\frac{x}{\alpha})$, but 1/4 times the base width. Therefore, its area is 1/4 times as that of area of $\delta(x)$ or 1/4.

SOL 1.3.30 Option (B) is correct.

This is a triangle with the same height as $\delta(x)$ but 1/6 times the base width. The fact that the factor is -6 instead of 6, just, means that the triangle is reversed in time which does not change its shape or area. Thus its area is 1/6 times as that of $\delta(x)$ or 1/6. The area of function

$$\delta(bx) = \lim_{a\to 0}\frac{1}{a}\text{tri}\left(\frac{bx}{a}\right),\quad a > 0 \text{ is } \frac{1}{|b|}$$

SOL 1.3.31 Option (C) is correct.

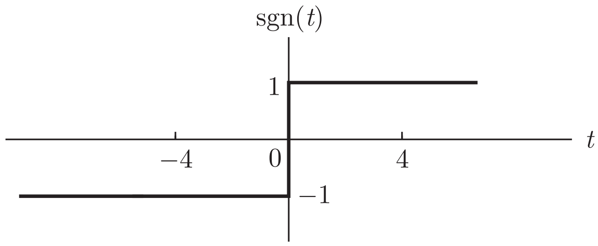
$$x(t) = 2\text{tri}\,[2\,(t-1)] + 6\text{rect}\left(\frac{t}{4}\right)$$

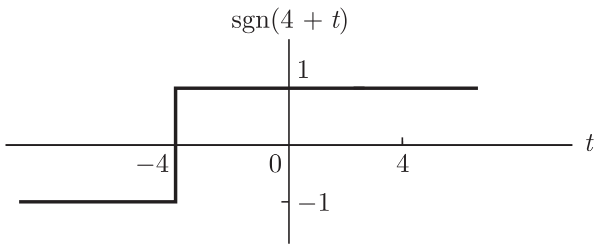
$$x\left(\frac{3}{2}\right) = 2\text{tri}\left[2\left(\frac{3}{2}-1\right)\right] + 6\text{rect}\left(\frac{3}{8}\right)$$

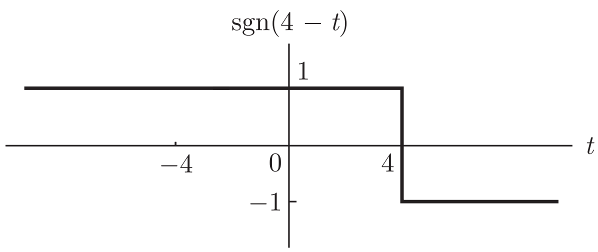
$$= 2\text{tri}\,(1) + 6\text{rect}\left(\frac{3}{8}\right) = 2\,[1-(1)] + 6 = 6$$

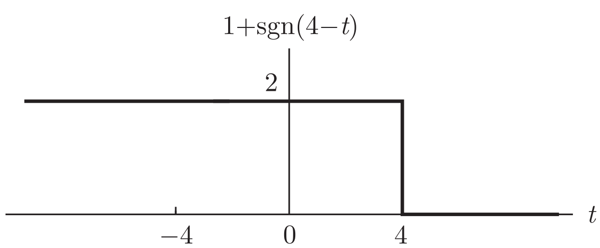
SOL 1.3.32 Option (A) is correct.

The figure is as shown below :







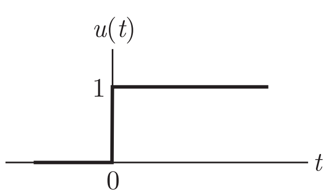


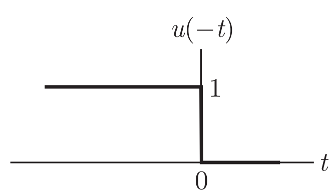
SOL 1.3.33 Option (D) is correct.

$v(t)$ is sum of 3 unit step signal starting from 1, 2, and 3, all signal ends at 4.

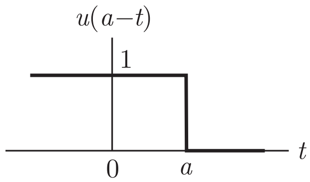
SOL 1.3.34 Option (B) is correct.

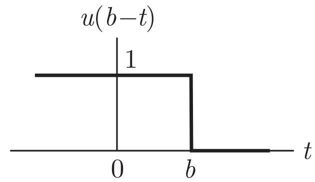
Unit step function $u(t)$ and its folded version $u(-t)$ are shown in the figures below



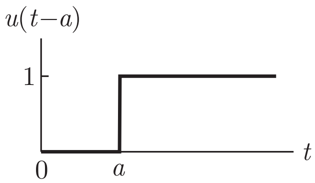


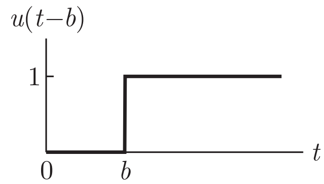
Now, by shifting $u(-t)$ to the right by a units and b units, we get $u(a-t)$ and $u(b-t)$ respectively.





Similarly, by shifting $u(t)$ to the the right by a units and b units, we get $u(t-a)$ and $u(t-b)$.





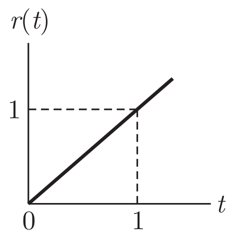
From the above graphs, we can see that

$$v(t) = u(t-a) - u(t-b)$$

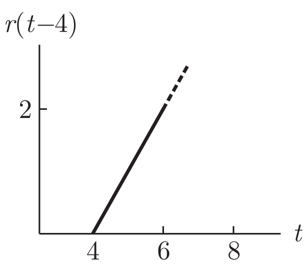
and, $$v(t) = u(b-t) \times u(t-a)$$

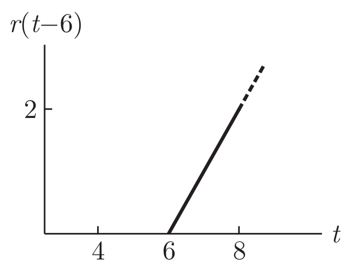
SOL 1.3.35 Option (B) is correct.

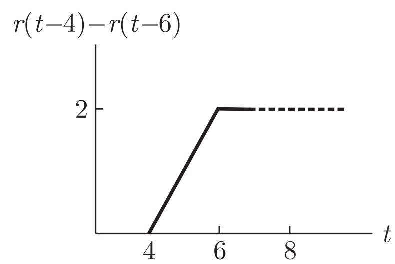
The ramp function is shown as



Signal $r(t-4)$ and $r(t-6)$ are obtained by shifting $r(t)$ towards right by 4 units and 6 units respectively. Now we subtract $r(t-6)$ from $r(t-4)$ to get $x(t)$.







$$x(t) = r(t-4) - r(t-6)$$

Alternate Method :

We have $$r(t-4) = \begin{cases} t-4, & t > 4 \\ 0, & t < 4 \end{cases}$$

and $$r(t-6) = \begin{cases} t-6, & t > 6 \\ 0, & t < 6 \end{cases}$$

Now $$r(t-4)-r(t-6)=\begin{cases} t-4, & 4<t<6 \\ t-4-t+6, & t>6 \\ 0, & t<4 \end{cases}$$

$$=\begin{cases} t-4, & 4<t<6 \\ 2, & t>6 \\ 0, & t<4 \end{cases}$$

SOL 1.3.36 Option (C) is correct.

To obtain the expression for $x(t)$, we note the change in amplitude and slope at different instants of time and write expression for each change. The steps are as follows :

1. At $t=0$, the function steps from 0 to 3, for a change in amplitude of 3. Also the slope of function changes from 0 to -3, for a change in slope of -3; so we write

$$x_1(t) = (3-0)\,u(t-0) + (-3-0)\,(t-0)\,u(t-0)$$
$$= 3u(t) - 3tu(t) = 3\,(1-t)\,u(t)$$

2. At $t=1$, the function steps from 0 to 1.5, for a change in amplitude of 1.5. Also the slope of function changes from -3 to -1.5, for a change in slope of 1.5; so we write

$$x_2(t) = 1.5u(t-1) + 1.5\,(t-1)\,u(t-1)$$
$$= 1.5u(t-1) + 1.5tu(t-1) - 1.5u(t-1)$$
$$= 1.5tu(t-1)$$

3. At $t=3$, the function steps up from -1.5 to 0, for a change in amplitude of 1.5. Also the slope of function changes from -1.5 to 0, for a change in slope of 1.5; so we write

$$x_3(t) = 1.5u(t-3) + 1.5\,(t-3)\,u(t-3)$$
$$= 1.5u(t-3) + 1.5tu(t-3) - 4.5u(t-3)$$
$$= 1.5tu(t-3) - 3u(t-3)$$
$$= 1.5\,(t-2)\,u(t-3)$$

Hence the equation for $x(t)$ is

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$
$$= 3\,(1-t)\,u(t) + 1.5tu(t-1) + 1.5\,(t-2)\,u(t-3)$$

SOL 1.3.37 Option (A) is correct.

To obtain the waveform for $x(t)$, we observe change in magnitude of unit step signals at different instants of time.

1. At $t=-1$, a step with magnitude 1 is added, so magnitude at $t=-1$ is 1.
2. At $t=1$, another step of magnitude -2 is added, so net amplitude becomes $(1-2)=-1$
3. At $t=3$, a step of magnitude 1 is added which causes net magnitude $(-1+1)=0$

Alternate Method :

From the expression we get

For $-1 < t < 1$, $x(t) = 1$

For $1 < t < 3$, $x(t) = -1$

For $t > 3$, $x(t) = 0$

SOL 1.3.38 Option (D) is correct.

Rearranging the given expression

$$x(t) = -2u(t+2) + u(t+1) + u(t)$$

The sketch of $x(t)$ is obtained using following steps :

1. At $t = -2$, a step of magnitude -2 is added, so magnitude at $t = -2$ is -2
2. At $t = -1$, another step of magnitude 1 is added which causes net magnitude to become $(-2+1) = -1$
3. At $t = 0$, another step of magnitude 1 is added, the net amplitude now becomes $(-1+1) = 0$.

Alternate Method:

For $-2 < t < 1$, $x(t) = -2$

For $-1 < t < 0$, $x(t) = -1$

For $0 < t$, $x(t) = 0$

SOL 1.3.39 Option (B) is correct.

By observing both the change in amplitude and change in slope, we get $x(t)$ as following :

1. At $t = 1$, a ramp of slope 2 is added, so the net slope of function becomes $(0+2) = 2$
2. At $t = 2$, a ramp of slope -2 is added which causes net slope to becomes $(2-2) = 0$
3. At $t = 3$, another ramp of slope 2 is added, now net slope of function becomes $(0+2) = 2$

Alternate Method :

For $1 < t < 2$, $x(t) = 2(t-1)$

For $2 < t < 3$, $x(t) = 2$

For $3 < t$, $x(t) = 2t - 2$

SOL 1.3.40 Option (D) is correct.

Rewriting the $x(t)$ as below

$$x(t) = -tu(t) + (t-1)u(t-1) + 2u(t-1) - u(t-2)$$

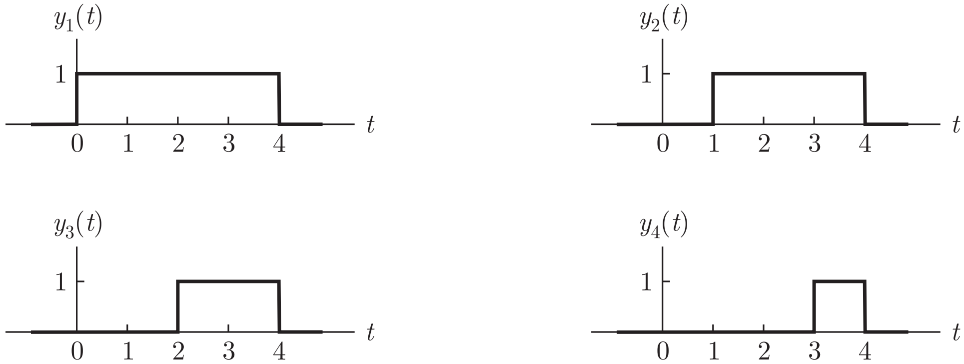
1. At $t = 0$, a ramp of slope -1 is added.
2. At $t = 1$, another ramp of slope 1 is added, so net slope at this instant becomes $(-1+1) = 0$
3. At $t = 1$, a step of amplitude 2 is added, so amplitude of $x(t)$ becomes

$(-1+2)=1$

4. At $t=2$ another step of amplitude -1 is added which causes net amplitude to become $(1-1)=0$

SOL 1.3.41 Option (A) is correct.

We may represent $y(t)$ as the superposition of 4 rectangular pulses as follows



$$y(t)=y_1(t)+y_2(t)+y_3(t)+y_4(t)$$

$y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$ are the time shifted and time scaled version of function $x(t)$ with different factors.

In general $\quad y_i(t)=x(a_i t-b_i) \qquad i=1,2,3,4$

$$y_1(t)=x(a_1 t-b)$$

For $t=0$, $\quad y_1(0)=x(a_1\times 0-b_1)=x(-1)$

$\Rightarrow \quad a_1\times 0-b_1=-1$

$$b_1=1$$

For $t=4$, $\quad y_1(4)=x(a_1\times 4-b_1)=x(1)$

$\Rightarrow \quad a_1\times 4-b_1=1$

$$4a_1=1+b_1\Rightarrow a_1=1/2$$

$$y_1(t)=x\left(\frac{1}{2}t-1\right)$$

$$y_2(t)=x(a_2 t-b_2)$$

For $t=1$, $\quad y_2(1)=x(a_2\times 1-b_2)=x(-1)$

$\Rightarrow \quad a_2-b_2=-1 \qquad$ ...(i)

For $t=4$, $\quad y_2(4)=x(a_2\times 4-b_2)=x(1)$

$\Rightarrow \quad 4a_2-b_2=1 \qquad$ ...(ii)

Solving equation (i) and (ii), we get $a=2/3$ and $b=5/3$

Thus, $\quad y_2(t)=x\left(\frac{2}{3}t-\frac{5}{3}\right)$

Similarly, we can obtain $y_3(t)$ and $y_4(t)$ also

$$y_3(t)=x(t-3)$$

$$y_4(t)=x(2t-7)$$

Accordingly, we may express the staircase signal $y(t)$ in terms of the rectangular pulses $x(t)$ as follows:

$$y(t) = x\left(\frac{1}{2}t - 1\right) + x\left(\frac{2}{3}t - \frac{5}{3}\right) + x(t-3) + x(2t-7)$$

SOL 1.3.42 Option (B) is correct.

$x_1(t)$ can be obtained using following methodology

1. At $t = 0$, slope changes from 0 to 2, so we write

$$x_1'(t) = 2tu(t)$$

2. At $t = 1$, slope change from 2 to -2 for a change of -4 in slope; so we write

$$x_1''(t) = -4(t-1)\,u(t-1)$$

3. At $t = 2$, slope changes from -2 to 0 for a change of 2 in slope; so we write

$$x_1'''(t) = 2(t-2)\,u(t-2)$$

Thus,

$$\begin{aligned} x(t) &= x_1'(t) + x_1''(t) + x_1'''(t) \\ &= 2tu(t) - 4(t-1)\,u(t-1) + 2(t-2)\,u(t-2) \end{aligned}$$

SOL 1.3.43 Option (B) is correct.

The expression for periodic waveform is

$$x(t) = \sum_{k=-\infty}^{\infty} x_1(t - kT_0)$$

Here, $T_0 = 2$, therefore

$$x(t) = \sum_{k=-\infty}^{\infty} x_1(t - 2k)$$

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SOLUTIONS 1.4

SOL 1.4.1 Option (D) is correct.

A signal conveys information on the nature of physical phenomenon.

SOL 1.4.2 Option (A) is correct.

$$x(t) = \cos(1.2\pi t) + \cos(2\pi t) + \cos(2.8\pi t)$$

Frequency of $\cos(1.2\pi t)$, $f_1 = 0.6\text{ Hz}$ $\qquad 2\pi f_1 = 1.2\pi$

Frequency of $\cos(2\pi t)$, $f_2 = 1\text{ Hz}$ $\qquad 2\pi f_2 = 1.2\pi$

Frequency of $\cos(2.8\pi t)$, $f_3 = 1.4\text{ Hz}$ $\qquad 2\pi f_3 = 2.8\pi t$

Fundamental Frequency of $x(t)$ will be greatest common divisor of f_1, f_2, f_3

$$\begin{aligned} f &= \text{GCD}(f_1, f_2, f_3) \\ &= 0.2\text{ Hz} \end{aligned}$$

SOL 1.4.3 Option (A) is correct.

We have
$$\begin{aligned} x(t) &= \cos(200\pi t) + 0.5\cos(40\pi t)\cos(200\pi t) \\ &= \cos(200\pi t) + \frac{1}{4}\cos 240\pi t + \frac{1}{4}\cos(360\pi t) \end{aligned}$$

Fundamental frequency of $(\cos 200\pi t)$, $f_1 = 100\text{ Hz}$ $\qquad 2\pi f_1 = 200\pi$

Fundamental frequency of $(\cos 240\pi t)$, $f_2 = 120\text{ Hz}$ $\qquad 2\pi f_2 = 240\pi$

Fundamental frequency of $(\cos 360\pi t)$, $f_3 = 180\text{ Hz}$ $\qquad 2\pi f_3 = 360\pi t$

Fundamental frequency of $x(t)$ is greatest common devisor of f_1, f_2 and f_3, i. e.

$$f = \text{GCD}(f_1, f_2, f_3) = 20\text{ Hz}$$

SOL 1.4.4 Option (C) is correct.

$$x(t) = 2\sin(2\pi t) + 3\sin(3\pi t)$$

Period of $\sin(2\pi t)$, $\qquad T_1 = \frac{2\pi}{2\pi} = 1\text{ sec}$

Period of $\sin(3\pi t)$, $\qquad T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}\text{ sec}$

Ratio $\qquad \frac{T_1}{T_2} = \frac{m}{n} = \frac{1}{(2/3)} = \frac{3}{2}$

Period of $x(t)$, $\qquad T = \text{LCM}\left(1, \frac{2}{3}\right) = 2$

SOL 1.4.5 Option (B) is correct.

We have $\qquad f(t) = \cos\left[\frac{\pi}{4}(t-1)\right]$

Period of $f(t)$, $\qquad T = \frac{2\pi}{\omega} = \frac{2\pi}{(\pi/4)} = 8\text{ sec}$

SOL 1.4.6 Option (A) is correct.

$$x_1(t) = 2\sin\pi t + \cos 4\pi t$$

Period of $\sin\pi t$, $T_{11} = \frac{2\pi}{\pi} = 2$

Period of $\cos 4\pi t$, $T_{12} = \frac{2\pi}{4\pi} = \frac{1}{2}$

$$\frac{T_{11}}{T_{12}} = \frac{2}{(1/2)} = 4 \text{ (rational)}$$

Since ratio of T_{11} and T_{12} is rational, $x_1(t)$ is periodic.

$$x_2(t) = \sin 5\pi t + 3\sin 13\pi t$$

Period of $\sin 5\pi t$, $T_{21} = \frac{2\pi}{5\pi} = \frac{2}{5}$

Period of $\sin 13\pi t$, $T_{22} = \frac{2\pi}{13\pi} = \frac{2}{13}$

$$\frac{T_{21}}{T_{22}} = \frac{(2/5)}{(2/13)} = \frac{13}{5} \text{ (rational)}$$

Since ratio of T_{21} and T_{21} is rational, $x_2(t)$ is also periodic.

SOL 1.4.7 Option (B) is correct.

The sum of two sinusoids is periodic if ratio of their periods is rational.

SOL 1.4.8 Option (A) is correct.

A signal is said to be periodic if it repeats at regular interval. If $x(t)$ is periodic with period T_0 it must satisfies.

$$x(t + T_0) = x(t)$$

SOL 1.4.9 Option (B) is correct.

We have $x_1(t) = e^{j20t}$

Period of $x_1(t)$, $T_1 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$x_2(t) = e^{-(2+j)t}$$

Since, $\frac{2\pi}{(2+j)}$ is not rational, so $x_2(t)$ is not periodic.

SOL 1.4.10 Option (A) is correct.

(A) $x_1(t) = \sin(10\pi t) + \sin(20\pi t)$

Period of $\sin(10\pi t)$, $T_{11} = \frac{2\pi}{10\pi} = \frac{1}{5}$

Period of $\sin(20\pi t)$, $T_{12} = \frac{2\pi}{20\pi} = \frac{1}{10}$

Ratio $\frac{T_{11}}{T_{12}} = \frac{1/5}{1/10} = 2 \text{ (rational)}$

Since ration of T_{11} and T_{12} is rational, $x_1(t)$ is periodic.

(B) $x_2(t) = \sin(10t) + \sin(20\pi t)$

Period of $\sin(10t)$, $T_{21} = \frac{2\pi}{10} = \frac{\pi}{5}$

Period of $\sin(20\pi t)$, $T_{22} = \frac{2\pi}{20\pi} = \frac{1}{10}$

Ratio, $\frac{T_{21}}{T_{22}} = \frac{\pi/5}{1/10} = 2\pi$ (not rational)

Since T_{21}/T_{22} is not rational, $x_2(t)$ is not periodic.
Similarly, we can check for option (C) and (D) also. Both are aperiodic.

SOL 1.4.11 Option (D) is correct.

Period of $x(t)$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{0.8\pi} = 2.5\,\text{sec}$

SOL 1.4.12 Option (D) is correct.

$$x(t) = x_1(t) + jx_2(t)$$

A complex valued signal always possess conjugate symmetry.

SOL 1.4.13 Option (A) is correct.

$$\Psi(t) = f(t) + f(-t)$$
$$\Psi(-t) = f(-t) + f(t)$$

Since $\Psi(t) = \Psi(-t)$ Thus $\Psi(t)$ is an even function.

SOL 1.4.14 Option (B) is correct.

We have $x(t) = A\cos(\omega t + \phi)$

We know that most of the periodic signals are power signal. $x(t)$ is also a periodic signal and has finite power.

$$p_x = \frac{A^2}{2}$$

SOL 1.4.15 Option (D) is correct.

Average power of signal is given by

$$P = \lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt$$

Note : If $x(t)$ is periodic, then T has finite value and above expression becomes as

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt = \frac{1}{T}\int_0^T |x(t)|^2\,dt$$

SOL 1.4.16 Option (C) is correct.

Energy of a power signal is infinite while the power of an energy signal is zero.

SOL 1.4.17 Option (A) is correct.

$$s(t) = 8\cos\left(\frac{\pi}{2} - 20\pi t\right) + 4\sin 15\pi t$$
$$= 8\sin 20\pi t + 4\sin 15\pi t$$

Here $A_1 = 8$ and $A_2 = 4$. Thus power is

$$P = \frac{A_1^2}{2} + \frac{A_2^2}{2} = \frac{8^2}{2} + \frac{4^2}{2} = 40$$

SOL 1.4.18 Option (B) is correct.

A bounded signal always possesses some finite energy.

$$E = \int_{-t_0}^{t_0} |g(t)^2|\,dt < \infty$$

SOL 1.4.19 Option (B) is correct.

Let E be the energy of $f(t)$ and E_1 be the energy of $f(2t)$, then

$$E = \int_{-\infty}^{\infty} [f(t)]^2\, dt$$

and

$$E_1 = \int_{-\infty}^{\infty} [f(2t)]^2\, dt$$

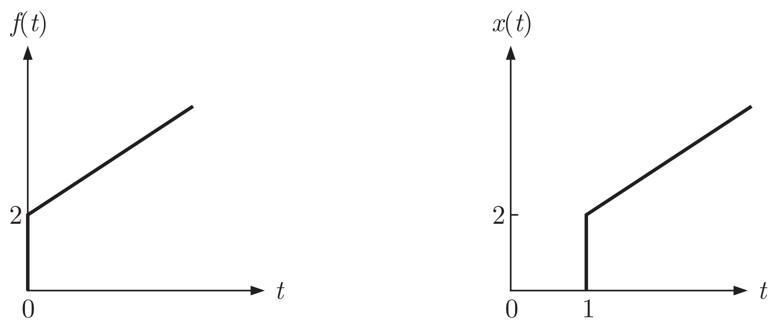
Substituting $2t = p$ we get

$$E_1 = \int_{-\infty}^{\infty} [f(p)]^2 \frac{dp}{2} = \frac{1}{2}\int_{-\infty}^{\infty} [f(p)]^2\, dp = \frac{E}{2}$$

SOL 1.4.20 Option (C) is correct.

If a function $f(t)$ is shifted to right side by t_0 units, then the shifted function is expressed as $f(t - t_0)\, u(t - t_0)$.

Let, $$f(t) = t + 2$$



$$x(t) = f(t-1)\, u(t-1)$$

If we write, $x(t) = f(t)\, u(t-1)$

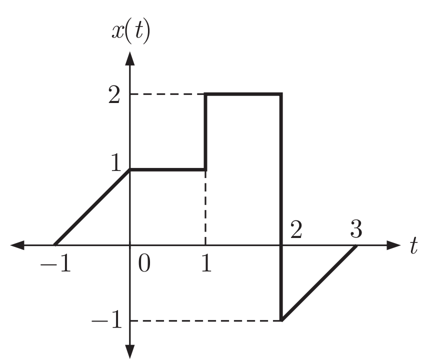
For $t = 0$ $x(0) = f(0) = 2$

But, $x(0) = 0$ (In the graph)

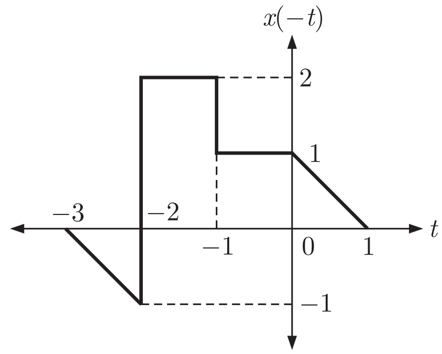
So $f(t)\, u(t - t_0)$ is not correct expression for shifted signal.

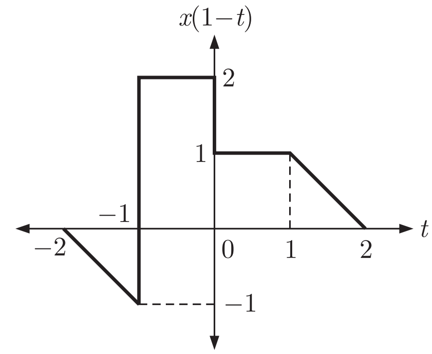
SOL 1.4.21 Option (A) is correct.

The plot of given signal $x(t)$ is shown below



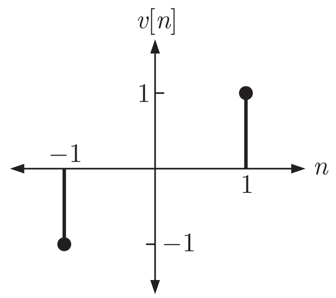
First reflect the signal about the vertical axis to obtain $x(-t)$. Then shift $x(-t)$ towards right by 1 unit to get $x(-t+1)$. Both operation is shown below

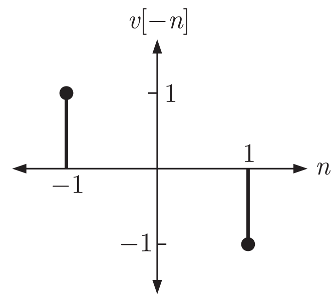




SOL 1.4.22 Option (A) is correct.

$v[n]$ and $v[-n]$ is drawn as





$$y[n] = v[n] + v[-n]$$
$$= 0, \text{ for all } n$$

SOL 1.4.23 Option (B) is correct.

Product property of impulse function

$$f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$$

For $t_0 = 0$, $\quad f(t)\,\delta(t) = f(0)\,\delta(t)$

Shifting property of impulse function

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$$

Area under Impulse function is unity.

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

For detailed discussion on properties of unit impulse function, refer page 32 of the book **GATE GUIDE Signals & Systems** by the same authors

SOL 1.4.24 Option (D) is correct.

Dirac delta function $\delta(t)$ is defined at $t = 0$ and it has infinite value a $t = 0$. The area of dirac delta function is unity.

SOL 1.4.25 Option (A) is correct.

We know that $\delta(t)\,x(t) = x(0)\,\delta(t)$ and $\int_{-\infty}^{\infty}\delta(t) = 1$

Let $x(t) = \cos\left(\frac{3}{2}t\right)$, then $x(0) = 1$

Now $\int_{-\infty}^{\infty}\delta(t)\,x(t) = \int_{-\infty}^{\infty}x(0)\,\delta(t)\,dt = \int_{-\infty}^{\infty}\delta(t)\,dt = 1$

SOL 1.4.26 Option (B) is correct.

We know that

$$\int_{-\infty}^{\infty}x(t)\,\delta(t-t_0) = x(t_0)$$

so

$$\int_{-\infty}^{\infty}\delta\left(t-\frac{\pi}{6}\right)6\sin(t)\,dt = 6\sin(t)\big|_{t=\pi/6} \qquad \text{Here } x(t) = 6\sin t,\ t_0 = \frac{\pi}{6}$$

$$= 6\sin\left(\frac{\pi}{6}\right)$$

$$= 6\times\frac{1}{2} = 3$$

SOL 1.4.27 Option (A) is correct.

$$x(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}t^2 e^{-t^2/2}\delta(1-2t)\,dt$$

Let, $1-2t = \alpha \Rightarrow t = \left(\frac{\alpha+1}{2}\right)$ and $dt = -\frac{1}{2}d\alpha$

Now

$$x(t) = \frac{1}{\sqrt{2\pi}}\int_{\infty}^{-\infty}\left(\frac{\alpha+1}{2}\right)^2 e^{-\frac{1}{2}\left(\frac{\alpha+1}{2}\right)^2}\delta(\alpha)\left(-\frac{1}{2}d\alpha\right)$$

$$= \frac{1}{2\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(\frac{\alpha+1}{2}\right)^2 e^{-\frac{1}{2}\left(\frac{\alpha+1}{2}\right)^2}\delta(\alpha)\,d\alpha \qquad \int_{-\infty}^{\infty}f(t)\,\delta(t)\,dt = f(0)$$

$$= \frac{1}{2\sqrt{2\pi}}\left(\frac{\alpha+1}{2}\right)^2 e^{-\frac{1}{2}\left(\frac{\alpha+1}{2}\right)^2}\Big|_{\alpha=0}$$

$$= \frac{1}{2\sqrt{2\pi}}\left(\frac{1}{2}\right)^2 e^{-\frac{1}{2}\left(\frac{1}{2}\right)^2} = \frac{1}{8\sqrt{2\pi}}e^{-\frac{1}{8}}$$

SOL 1.4.28 Option (B) is correct.

$$\int_0^t\int_0^t u(t)\,dt = \int_0^t tu(t)\,dt = \frac{t^2}{2}, \qquad \text{(Parabola)}$$

SOL 1.4.29 Option (A) is correct.

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

and

$$x_o(t) = \frac{x(t)-x(-t)}{2}$$

Here

$$g(t) = u(t)$$

Thus

$$x_e(t) = \frac{u(t)+u(-t)}{2} = \frac{1}{2}$$

$$x_o(t) = \frac{u(t)-u(-t)}{2} = \frac{x(t)}{2}$$

SOL 1.4.30 Option (D) is correct.

At $t=1$, signal steps up from $0\to 1$, so

$$v_1(t) = (1-0)\,u(t-1) = u(t-1)$$

At $t=2$, signal steps up from $1\to 2$, so

$$v_2(t) = (2-1)\,u(t-2) = u(t-2)$$

At $t=3$, signal steps up from $2\to 3$, so

$$v_3(t) = (3-2)\,u(t-3) = u(t-3)$$

At $t = 4$, signal steps down from $3 \to 0$, so

$$v_4(t) = (0-3)\,u(t-4) = -3u(t-4)$$

$$v(t) = v_1(t) + v_2(t) + v_3(t) + v_4(t)$$

$$= u(t-1) + u(t-2) + u(t-3) - 3u(t-4)$$

For detailed discussion please refer to methodology of section 1.6 of the book **GATE GUIDE Signals & Systems** by same authors.

SOL 1.4.31 Option (A) is correct.

We know that ramp function is obtained by double differentiation of impulse function.

$$\underset{\text{(Ramp)}}{r(t)} \xrightarrow{\text{differentiation}} \underset{\text{(Step)}}{u(t)} \xrightarrow{\text{differentiation}} \underset{\text{(Impulse)}}{\delta(t)}$$

$$tu(t) \xrightarrow{\text{differentiation}} u(t) \xrightarrow{\text{differentiation}} \delta(t)$$

Given Function is

$$f(t) = -\delta(t-1) - \delta(t-2) + \delta(t-3) + \delta(t-4) - \delta(t-5) + 2\delta(t-6) - \delta(t-7)$$

In-terms of ramp function

$$f(t) = -tu(t-1) - tu(t-2) + tu(t-3) + tu(t-4) - tu(t-5) + 2tu(t-6) - tu(t-7)$$

SOL 1.4.32 Option (B) is correct.

(A) $v(t) = u(t-1) - u(t-3)$ (A $\to$ 3)

(B) $v(t) = \lim_{a \to 0} \delta(t-1)$ (B $\to$ 4)

(C) $v(t) = u(t+1)$ (C $\to$ 1)

(D) $v(t) = u(t) - 2u(t-1) + 2u(t-2) - 2u(t-3) + \ldots$ (D $\to$ 2)

SOL 1.4.33 Option (C) is correct.

At $t = 0$, $f(t)$ step up from $0 \to 1$, so we write

$$f_1(t) = (1-0)\,u(t-0) = u(t)$$

At $t = 1$, $f(t)$ steps up from $1 \to 2$, so we write

$$f_2(t) = (2-1)\,u(t-1) = u(t-1)$$

At $t = 2$ slope changes from $0 \to 1$ so we write

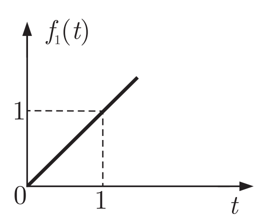
$$f_3(t) = (1-0)(t-2)\,u(t-2)$$

Now, $$f(t) = f_1(t) + f_2(t) + f_3(t)$$

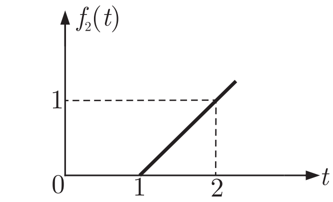
$$= u(t) + u(t-1) + (t-2)\,u(t-2)$$

For detailed discussion please refer to methodology of section 1.6 on page 37, given in the book **GATE GUIDE Signals & Systems** by the same authors.

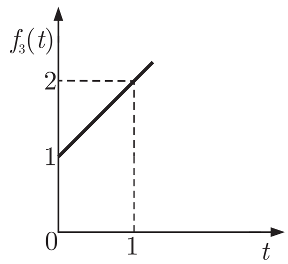
SOL 1.4.34 Option (B) is correct.



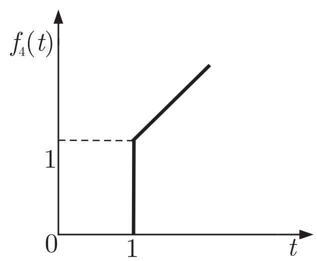
$$f_1(t) = tu(t) \to \text{option}(3)$$



$$f_2(t) = \text{shift } f_1(t) \text{ by } 1 \text{ unit}$$
$$= (t-1)\,u(t-1) \to \text{option}\,(6)$$



$$f_3(t) = \; tu(t) + u(t)$$
$$= (t+1)\,u(t) \to \text{Option}\,(4)$$



$$f_4(t) = tu(t+1) \to \text{option}\,(1)$$

SOL 1.4.35 Option (D) is correct.

At $t = 3$ slope changes from $0 \to 2$, so we write

$$v_1(t) = (2-0)\,(t-3)\,u(t-3)$$
$$= (2t-6)\,u(t-3)$$

at $t = 4$, $v(t)$ becomes zero, so

$$v(t) = (2t-6)\,[u(t-3) - u(t-4)]$$

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